Factions in Rousseau's Du Contrat Social *and federal representation*

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1. *Two questions*

Consider the following two seemingly unrelated questions. First, why does Rousseau (1993 [1762]) believe that the formation of *factions* or *partial associations* is not conducive to the general will in *Du Contrat Social*, II, 3? Second, why do federal assemblies typically strive for some form of degressive proportionality, i.e. a balance between equal and proportional representation, for the countries in the federation? We will show that there is a surprising connection between these questions.

We turn to our first question. It is often thought that Rousseau's opposition to factions can be interpreted in reference to the Condorcet Jury Theorem (e.g. Grofman and Feld 1988). The Condorcet Jury Theorem states that if voters are more likely to be right on some issue than not and cast their votes independently, then the chance that the majority is correct converges to one as the number of voters goes to infinity. This interpretation is supported by Rousseau's claim that 'If, when a sufficiently informed people deliberates, the citizens *were to have no communication between themselves*, the general will would always result from the large number of small differences' (II, 3, emphasis added). If voters form factions, then they no longer vote independently. They become subject to opinion leaders and the epistemic advantage of consulting a large number of voters is lost. Or, as Grofman and Feld (1988: 571) put it, the effective number of votes is reduced, which in turn reduces the probability that the majority is correct.

This interpretation is tempting, but it is inconsistent with some crucial textual evidence. Rousseau writes, 'If there are partial societies, it is necessary to multiply their number and to prevent inequality from existing among them ... These precautions are the only good ones for ensuring that the general will is always enlightened' (II, 3). Now if Rousseau is interpreted in terms of the Condorcet Jury Theorem in the way described above, then multiplying factions only makes things worse. The effective number of votes is further reduced. Thus, even if factions form, one should retain the independence between the remaining parties as much as possible to safeguard whatever epistemic advantage the majority vote still has. An alternative interpretation of Rousseau's opposition to factions is called for.

We turn to our second question. Consider a federation of countries with a decision-making assembly in which each country casts a block vote. On

Analysis 67.1, January 2007, pp. 12–20.

equal representation, the vote of each country has the same weight. On proportional representation, the weights of the votes are proportional to the countries' population sizes. In between these extremes, we can let the weights increase as a function of population sizes, but smaller countries receive greater weights and larger countries receive lesser weights than proportionality would warrant. Such weightings are called *degressively proportional* weightings. How can degressive proportionality be justified? And does this justification provide us any guidance as to how these weights should be set?

A correct understanding of Rousseau's misgivings about the formation of factions will hold the key to these questions.

2. *Factions and degressive proportionality*

Imagine a federation in which people vote on proposals. A particular proposal imposes costs on some people and yields benefits for other people if it is accepted. Assume that each person has the same chances of incurring certain costs or receiving certain benefits from a proposal and that costs or benefits bestowed on one person do not tell us anything about costs or benefits bestowed on other persons. Each person has one vote and votes for all and only proposals that are beneficial to her. Proposals are accepted by majority vote. This is an unproblematic state of affairs: each person has precisely the same expected utility from a random proposal.

But now some cunning people realize that they can gain advantages by forming a faction. They strike the following deal. When a proposal comes to the table, they will not automatically vote for or against the proposal according to whether they see costs or benefits coming their way but, rather, they will confer with each other. They will vote for the proposal if and only if the *average* utility is positive for the persons in the faction. By doing so, the expected utility from a random proposal for factionists typically goes up and the expected utility for isolated people typically goes down. Voting in factions is typically beneficial for factionists and detrimental for isolated people.

Here is a simple example with a three-person society. Suppose that there is a 1/3 chance that a proposal will yield a benefit of +1 for an arbitrary person, a 1/3 chance that it will yield a cost of −1, and a 1/3 chance that it will not affect her at all, i.e. her utility is 0. A person will vote for the proposal just in case it yields a benefit and proposals are accepted by majority vote. Then the proposal is accepted for the profiles in Table 1. Every profile has $(1/3)^3$ chance of occurring. Hence the expected utility of each person $i = \text{A, B, C}$ is $\text{E}[U_i] = 4/27$. Now A and B decide to form a faction: they will vote for a proposal just in case the proposal has a positive average utility for them. The proposal is then accepted for the

Table 1: Voting profiles under which a motion is accepted, when there are no factions.

A	B	C
1	1	1
1	1	0
1	1	−1
1	0	1
1	−1	1
0	1	1
−1	1	1

Table 2: Voting profiles under which a motion is accepted, when A and B form a faction.

A	B	C
1	1	1
1	1	0
1	1	−1
1	0	1
0	1	1
0	1	0
1	0	0
0	1	−1
1	0	−1

Table 3: Expected utilities of factionists f and isolated persons i after multiplying the number of equal-sized factions.

	{A,...,I}	{{A,B,C}, D,...,I}	{{A,B,C},{D,E,F}, G,H,I}	{{A,B,C},{D,E,F}, {G,H,I}}
$E[U_f]$	–	0.116	0.0739	0.0762
$E[U_i]$	0.0569	0.0493	0.0691	–

profiles in Table 2. Note that the profiles in rows five and seven in Table 1 have dropped out and four more profiles are added. The expected utilities $E[U_A] = E[U_B]$ of the factionists increases to 6/27 and the expected utility $E[U_C]$ of the isolated person decreases to 0. So C will understandably look with disgruntlement at the cunning of A and B.

Now what happens when we follow Rousseau's advice and form additional factions? Take a nine-person society and let us do precisely what we did for the three-person society, except that we will form three-person factions and multiply the number of factions. We show results in Table 3. Before faction formation $E[U_i] = 0.0569$ for $i = A, \ldots, I$. After A, B and C form a faction, the expected utility of the factionists $f = A, B, C$ goes up to 0.116 and the expected utility of the isolated persons $i = D, \ldots, I$ goes down to 0.0493. Now D, E and F fight back by forming a faction and they raise their expected utility to 0.0739 and restore equality with A, B and C. Finally, when G, H and I form a faction, the equality in expected utility across the federation is restored at 0.0762.

As soon as the first faction forms, the equality between the expected utilities of the persons in the federation is lost. Rousseau disapproves, because 'every authentic act of the general will' is supposed to 'bind or favour all the citizens equally' (II, 4). However, if factions form, then we should 'multiply their number' and 'prevent inequality between them' (II, 3). Rousseau's 'should' is both of a prudential and moral nature. It is a prudential 'should' in that isolated people can often improve their situation by forming factions and raise their expected utility, thereby restoring equality with the expected utility of factionists. It is a moral 'should' in that equality in society is restored if society is partitioned in equal-sized factions.

So we have an interpretation that is consistent with Rousseau's advice. But factions do not form easily. There are transaction costs and even if factions do form, they may not be factions of equal sizes. The expected utilities of factionists in unequal-sized factions will typically vary as the entries in the Table 4 show.

Are there any other means of preventing the ill effects of partial faction-formation or unequal-sized faction formation? So far we have assumed that the number of votes for a faction equals the sum of the votes of the factionists, i.e. the factions have weights proportional to their sizes. In the following we will take the weights for the factions as variables. We will try to set the weights so as to counterbalance the effects of faction formation and to restore equality.

Suppose, now, that we succeed in assigning faction weights that do precisely this. Then we have also solved the problem of how to assign weights to countries in the federal assembly. For we can think of a federation of countries as a society in which cunning factionists have been at work and have created smaller and larger countries that will vote en bloc depending on whether a proposal is in the interest of the country or not. Under proportional weights, larger countries receive greater expected utility from proposals than smaller countries, just as in our nine-person society. Smaller countries may complain that they are only willing to remain in a federation in which proposals spread costs and benefits equally across the citizens of different countries in the federation. And this is indeed a feasible political ideal. They could follow Rousseau's advice

Table 4: Expected utilities of factionists in unequal-sized factions and of isolated people.

	{A,B,C}	{D,E},{F,G}	H,I
E[*U*] in {A,B,C},{D,E},{F,G}, H,I}	0.960	0.0579	0.0338

and factionalize further by starting to vote en bloc with other small countries. But such a process is unlikely to yield stable equal-sized voting blocs.

So another solution must be found to placate smaller countries. This is where degressive proportionality comes in. The weights should be made precisely so degressive that the expected utility of a proposal is equalized across the federation, as it was before factions started forming. How far should we pull back from proportional representation in order to realize the desideratum of equal expected utilities from proposals? To address this question we need to set up a more complex model.

3. *A model of faction formation*

Consider a federation consisting of N countries where each country i has m_i inhabitants. A proposal is a vector of utilities v_i^j, where v_i^j is the utility that a person j in country i will receive if the proposal is accepted. Utilities are scaled so that positive utilities represent benefits from a motion and negative utilities represent costs. We construct random variables V_i^js that are independently, identically and normally distributed with a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$. Countries vote as factions, i.e. a country i votes for a proposal just in case $v_i = (v_i^1 + ... + v_i^{m_i})/m_i > 0$. It is easy to show that the corresponding variables V_is are normally distributed with a constant $\mu = 0$ and with different standard deviations $\sigma_i = 1/\text{Sqrt}[m_i]$.

A decision rule assigns a certain weight to each country. If the proposal receives more than half of these weights in the vote then it passes. Subsequently, each person j from country i receives utility $u_i^j = 0$ if the proposal does not pass and $u_i^j = v_i^j$ if the proposal does pass. Call the corresponding random variables that yield the utilities after the decision rule has been applied U_i^j. We are interested in the expected utilities for the counties, $E[U_i^j]$. Since the $E[U_i^j]$s are identical for all persons j in the same country i, we denote them by $E[U_i]$.

The $E[U_i]$s will vary with the weights assigned to the countries. There is a family of weightings that is parameterized by a measure α ranging from 0 for equal representation and 1 for proportional representation. Let x_i be the relative population size of each country i in the federation and w_i the weight of each country in the federal assembly: $w_i(\alpha) = x_i^\alpha / \Sigma x_k^\alpha$. For $\alpha = 0$, $r_i = 1/n$ for the n countries in the federation and there is equal representation in the assembly. For $\alpha = 1$, $r_i = x_i$ and there is proportional representation in the assembly. Intermediate values of α yield models of representation of degressive proportionality. (Cf. Felsenthal and Machover 1998: 73 and Bovens and Hartmann 2006, Eq. (1).)

Our desideratum is to equalize the E[U_i]s by picking a particular model of degressive proportionality. We want to determine for what value of α the scatter of the E[U_i]s is minimal and we measure this scatter by the standard deviation of the E[U_i]s.

As a working example, we chose the European Union. Each country i is a faction consisting of m_i inhabitants. We calculate the E[U_i]s[1] and plot the standard deviation of the E[U_i]s as a function of α in Figure 1. The standard deviation is minimal for a value of α in the neighborhood of 1/2, i.e. when we set the weights of the countries roughly proportional to the square roots of the population sizes.

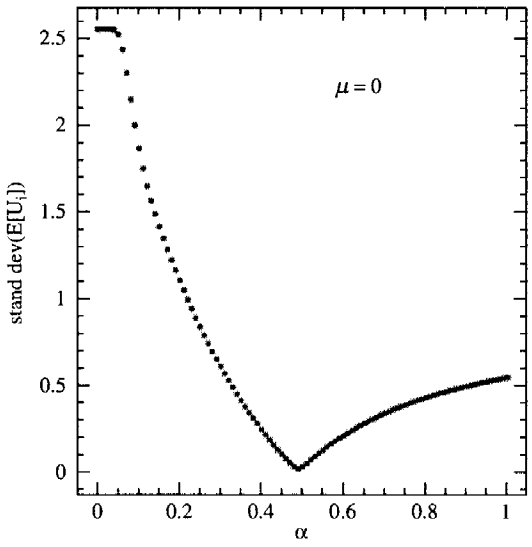

Figure 1. The standard deviation of the E[U_i]s as a function of α.

[1] We calculate the expected utilities E[U_i] in the following way. We define a function λ that maps positive utilities into 1 for a yes vote, and non-positive utilities into 0 for a no vote. Consider a proposal that would yield benefits or costs $v_1, \ldots, v_n$ to countries $1, \ldots, n$ if it were accepted. According to our model of faction formation, the voting profile for the proposal is $<\lambda(v_1), \ldots, \lambda(v_n)>$. Subsequently we define a function D that maps profiles for which the proposal is accepted into 1 and profiles for which the proposal is rejected into 0. Then the utility that country i gets out of the decision process equals $U_i = V_i D(<\lambda(V_1), \ldots, \lambda(V_n)>)$ and the expectation of the utility $E[U_i] = \int dv_1 \ldots \int dv_n p(v_1) \ldots p(v_n) v_i D(<v_1, \ldots, v_n>)$. (Beisbart et al. 2005: 401–2).

4. Discussion

Let us look at some passages in Rousseau in support of our interpretation. In II, 3 Rousseau thinks of the general will as the outcome of some kind of balancing act between conflicting private interests. There is the obscure definition of the general will: '... take away from [the particular wills] the pluses and the minuses that cancel each other out, and the general will remains as the sum of the differences'. In a footnote, he comments: '... the agreement of all interests is formed by opposition to that of each. If there were no different interests, the common interest would be barely felt, as it would encounter no obstacle; all would go on of its own accord, and politics would cease to be an art' (II, 3).

Subsequently, there is the quote that has been invoked to support the Condorcet-Jury-Theorem interpretation: 'If, when a sufficiently informed people deliberates, the citizens were to have no communication between themselves, the general will would always result from the large number of small differences' (II, 3). This passage can also be read to support our interpretation. We understand the 'no communication' clause in terms of the absence of faction formation. However, when there is faction formation, 'the will of each of these associations becomes general with respect to its members, and particular with respect to the state' and 'there are no longer as many voters as men, but only as many as there are associations' (II, 3). This quote introduces the relevance of voting and corresponds to our point that factions assess whether a proposal is in their common interest and then cast a bloc vote accordingly.

Finally, there is the equality condition: 'Every authentic act of the general will' is supposed to 'bind or favour all the citizens equally' (II, 4). As we have shown, equal expected utility from proposals is precisely what is lost through faction formation. So what can be done? Rousseau quotes Machiavelli who requires that 'a founder of a republic ... can at least see to it that there are no factions' (II, 3). Alternatively, Rousseau suggests that one should 'multiply the number [of factions] and prevent inequality between them' (II, 3). And indeed, either we can block factionalist voting or we can construct a society that consists of equal-sized factions to warrant that voting yields proposals that afford equal expected utility to each citizen.

However, in a federation, the countries voting as factions are already in place and they are not equal-sized. Small countries can start voting en bloc, but such agreements are unstable and representatives have a foremost responsibility to defend the interests of their own citizens. We have shown that there is yet another alternative. To restore equality between unequal-sized factions, we can adjust the weights of their votes. And that is precisely what is done in degressively proportional weightings for coun-

tries – voting as factions – in a federation. Furthermore, we have shown that to restore equality we should set weights proportional to the square root of the faction size.[2]

One may object that this could not possibly be what Rousseau meant, because we have pictured the voters as Benthamite voters, i.e. voters who have an eye for their private interests. They are willing to compromise for factional interests, but never do they become Rousseauian voters, i.e. voters who vote with an eye to the public interest. In response, we believe that *Du Contrat Social* is a rich and varied text and it is a more fruitful approach to try to acquire from passages what can be learned in the way of democratic theory rather than to strive for the most consistent interpretation of Rousseau's oeuvre. Our interpretation captures the perils of factionalization and at the same time makes sense of the injunction to counter existing factions with more factions, whereas the epistemic interpretation fails to do the latter.

It is no coincidence that our model makes the same recommendation as a well-known result from the voting-power literature (Penrose 1946 and 1952; Felsenthal and Machover 1998: 35–78). If we wish to give each person in a federation the same chance of being *doubly pivotal*, then, given certain assumptions, the representatives of each country should have voting weights proportional to the square root of the population sizes as well. A person is doubly pivotal, in that, had she voted differently, then her representative would have voted differently and had her representative voted differently, then the vote on the proposal in the assembly would have come out differently. It is easy to show that, given the assumptions of our model, the chance of *being doubly pivotal* is proportional to the chance of *getting one's way*, i.e. of the outcome of the vote coinciding with one's vote. Given the assumptions of our model, equalizing the chance of getting one's way – and hence of double pivotality – is tantamount to equalizing the expected utility of proposals. Hence, our result in §3 is substantially identical to Penrose's result. Nonetheless, the appeal to utilities and the dynamics of voting in factions

[2] Our model rests on the assumption that the random variables V_is are independently distributed. One might object that factions (or countries) are formed precisely because people's interests are aligned at least to some extent. Indeed, this is a valid objection to an idealization in our model. Now, if the interests of the persons in a country were *fully* aligned, then countries should be assigned one vote – i.e. we should institute a model of equal representation or $\alpha = 0$ – to preserve equality. The reality is somewhere between both extremes. There is some dependency between the interests of the persons that form a country but there is no full dependency. Hence a proper model of representation for the real world should strike a balance between equal representation and the square root rule.

makes it vivid why equality requires less than proportional representation and this is absent from Penrose's argument. Furthermore, our result is also of historical significance: It shows how the square root rule in the voting-power literature can be seen as a response to a problem first posed by Rousseau.[3]

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[3] We are grateful to Robert Sugden for suggesting the connection between factional voting and degressive proportionality, to Alex Voorhoeve for directing us towards Rousseau's discussion of factions, and to Alice Obrecht for comments.