Synthese (2006) 150:131–153
DOI 10.1007/s11229-006-0005-1

RESEARCH ARTICLE

Democratic answers to complex questions – an epistemic perspective

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Abstract This paper addresses a problem for theories of epistemic democracy. In a decision on a complex issue which can be decomposed into several parts, a collective can use different voting procedures: Either its members vote on each sub-question and the answers that gain majority support are used as premises for the conclusion on the main issue (*premise based-procedure, pbp*), or the vote is conducted on the main issue itself (*conclusion-based procedure, cbp*). The two procedures can lead to different results. We investigate which of these procedures is better as a truth-tracker, assuming that there exists a true answer to be reached. On the basis of the Condorcet jury theorem, we show that the *pbp* is universally superior if the objective is to reach truth for the right reasons. If one instead is after truth for whatever reasons, right or wrong, there will be cases in which the *cbp* is more reliable, even though, for the most part, the *pbp* still is to be preferred.

Keywords Discursive dilemma · Condorcet jury theorem · Judgment aggregation · Voting procedures · Epistemic democracy · Deliberative democracy · Pettit

Two voting procedures

Suppose a committee must take a stand on a complex issue, in which the decision depends on how several sub-questions are answered. The committee agrees which sub-questions should be posed. All questions are of the yes-or-no type and the main question is answered with "yes" if and only if each sub-question is answered with "yes". After discussion, the committee

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proceeds to a vote in which the majority rule is being applied. Two different voting procedures can be used. On one procedure, the committee members vote on each sub-question and the voting results then determine the committee's conclusion on the main issue. In other words, the vote is on each premise and the conclusion is accepted if and only if each of the premises is supported by a majority of the committee members. This *premise-based procedure* (or *pbp*, for short) can be contrasted with the *conclusion-based procedure* (*cbp*). On that procedure, the members directly vote on the conclusion, with the vote of each member being guided by her views on the relevant sub-questions.

To illustrate, suppose a company is considering purchasing a new item of equipment. The board is willing to approve the purchase if and only if the item meets certain safety standards and the purchase is economically feasible. The board can follow the premise-based procedure: it can separately vote on each factor (safety and feasibility, respectively) and then decide for the purchase if the item has got a majority with respect to each of the factors. On this procedure, the board never gets to vote on the conclusion, but only on the premises. Or the conclusion-based procedure can be followed: Each board member votes for the purchase if and only if she thinks all the requirements are met. The majority in this vote on the conclusion determines whether the purchase will be made. No vote is conducted on the premises.

These procedures are by no means equivalent. Within legal theory, this has been noticed in connection with jury votes (cf. Chapman, 1998a, b; Kornhauser, 1992a, b; Kornhauser & Sager, 1986, 1993). There may be a majority of voters supporting each premise, but if these majorities do not significantly overlap, there will be a majority against the conclusion.

Pettit (2001) connects the choice between the two procedures with general political theory, in particular, with the discussion of *deliberative democracy.* (This connection is also made in Brennan (2001). For a general characterization of deliberative democracy, cf. Elster, 1998.) Pettit mentions three requirements that are essential for a deliberative democratic regime. (i) All members of the group should vote. (ii) In voting, each person takes a considered stand on what decision is reasonable from the point of view of the goals of the group (rather than a stand from the point of view of her personal goals). (iii) Voting is preceded by a process of deliberation that takes place in an open dialogue between the group members. Furthermore, an important desideratum is that the collective decisions be *contestable*: It should be possible for the citizens to criticize democratic decisions by questioning their underlying reasons. The premise-based procedure makes such contestability much easier, as Pettit points out, since it gives the premises of an argument a democratic imprimatur and thus places them in a public arena. It thereby allows for the contestation of the conclusion by questioning the premises. The conclusion-based procedure, on the other hand, keeps the premises out of the public arena and hence such a democratic regime dodges accountability for the reasons behind its decisions. To this observation of Pettit, one might also add another consideration: Deliberative democracy puts a premium on collective deliberation and reasoning. A group that follows a premise-based procedure may be said to reason as a collective: It takes as a whole a stand on the premises and from there it moves to the conclusion. The conclusion-based procedure also contains or at least allows for some collective elements: It does involve reaching a collective decision and it may be preceded by some collective deliberation. Still, in contrast to the premise-based procedure, it lets the reasoning from the premises to the conclusion proceed on the individual level. Thus, even in this respect, the premise-based procedure more closely approximates the ideal of deliberative democracy.

However, the problem we want to examine concerns the relative advantages and disadvantages of the two procedures from the *epistemic* point of view. In some cases one can assume that the question before the committee has a *right answer*, which the committee is trying to reach. With regard to such cases, it is natural to ask: Is one of the two procedures better when it comes to tracking the truth? As it turns out, the answer to this query is not univocal. On the basis of Condorcet's jury theorem we shall show that the premise-based procedure is clearly superior if we want to reach truth for the *right reasons*, i.e. without making any mistakes on the road to the conclusion. However, if the goal instead is to reach truth for *whatever reasons*, right or wrong, there will be a range of cases in which using the conclusion-based procedure is more reliable. But, for the most part, the premise-based procedure will retain its superiority. In this respect, our results partly disconfirm the tentative conjectures that have been put forward in Pettit and Rabinowicz (2001).

The Condorcet jury theorem

It need not always be the case that there exists an independent truth, which can be tracked by a democratic voting procedure. In some contexts, the right decision is simply the decision that is reached by a legitimate political procedure. Still, it seems that such a purely "procedural" reading of right and wrong would quite often be inappropriate. For example, in many cases, the voters on the losing side might well consider the majority decision to be wrong, even if they are prepared to abide by it. What they object to is not the legitimacy of the decision-making process but its outcome. And the objections need not be framed in terms of their personal interests; they might well appeal to the goals of the collective. The minority voters might argue that the decision, however legitimate, was an incorrect one for the collective to take. If such views are reasonable, then it is meaningful to evaluate collective decision procedures from the epistemic perspective and compare their capacities as truth-trackers. "Epistemic" democrats take democracy to be especially valuable from such a truth-oriented perspective. (Cf. Estlund, 1990, 1993, 1997, 1998; List & Goodin, 2001. The label itself, "an epistemic theory of democracy", comes from Cohen, 1986.). Rousseau is often seen as a founding father of this approach to democracy. It is central in Rousseau's theory that voters express their views on the "general will" rather than their individual preferences. (see Rousseau, 1997 (1762), book 4, ch. 2. In a modified version this idea is retained by the deliberative democrats, who require the voters to express their opinions as to which decision is best from the point of view of the common goals of the collective.) At the same time, another French Enlightenment figure, Marquis de Condorcet, is given credit for the theorem that is meant to clarify democracy's epistemic advantage (cf. Condorcet, 1785, 279ff; for an English translation of the relevant passages, see McLean & Urken, 1995). This Condorcet Jury Theorem (CJT), in one of its versions, can be formulated as follows:

(CJT) Suppose there are n voters, with n being odd and greater than 1. For some p such that $1>p>.5$, suppose that each voter has a chance p of correctly assessing whether a proposition is true or false and that this chance is independent of whether the other voters' assessments are correct or not. Then the probability that the majority vote is a correct assessment of whether this proposition is true or false (i) is greater than p and (ii) converges to 1 as the number of voters increases to infinity.

If, on the other hand, each voter's chance p of being correct is lower than .5 (but still higher than 0), then the chance of the majority being correct is lower than p and decreases to 0 as the number of voters increases. If $p = .5$, the chance of the majority being correct will be .5, however much we increase the number of voters. Still, as long as the voters are even slightly reliable, i.e. if their opinions are worth more than a random coin flip, the CJT says that the majority view will be more reliable than the opinion of a single voter.

There are three assumptions in this statement of the theorem that can readily be relaxed. There is (a) a constraint on the number of voters; (b) a symmetry assumption; and (c) an independence assumption.

(a) *The number of voters is odd.* The consequence (ii) in CJT also holds for even-numbered voters, but not the consequence (i). To see that (i) does not hold for even-numbered voters, set their number at 2. Then, given the other assumptions of the theorem, the chance that the majority of voters, i.e. both voters, are correct is p^2, which is smaller than p for $0 < p < 1$. However, for all even numbers $n > 2$, there exists a number $p(n) \in (.5, 1)$ such that (i) holds for any $p \in (p(n), 1)$. Furthermore, $p(n)$ is a decreasing function of the even numbers n and it asymptotically decreases to .5 as n approaches infinity. (See appendix 1.)

(b) *The voters are equally competent.* Each voter has the same chance p, where $.5 < p < 1$, of correctly assessing whether the proposition is true or not. The requirement that this level of competence is the same for all can be relaxed by assigning to each voter i a chance p_i so that their *average* competence of correctly assessing whether the proposition is true or not is contained in (.5, 1). (Cf. Borland, 1989; Grofman, Owen, & Feld, 1983; Owen, Grofman, & Feld, 1989)

(c) *The voters are independent as far the correctness of their assessments is concerned.* For any voter i, the chance that i's assessment of a given proposition is correct is independent of whether any of the other voters' assessments of this proposition are correct or not. Suppose we learn about some subset of the remainder of the voters whether they are or are not correct in their assessments, without learning anything about what votes they have cast. Then this newly acquired information is not supposed to affect the probability of voter i being correct.[1] It is easy to see that the theorem does not hold when the voters are voting en bloc: Suppose that one person casts an autonomous vote and has a chance p of being correct, while all the others simply duplicate his vote. Then the chance that the majority vote is correct is still precisely p. In general, the chance that the majority is correct decreases as the committee members are more influenced by how other members vote. However, the CJT still stands as long as the influence of opinion leaders is not too overwhelming (cf. Estlund, 1994).

To avoid computational complexity, we will conduct our investigation under the assumptions of an odd number of voters, equal competence and independent voters. Essentially the idea behind the CJT is simple. If the competence of each voter is independent of whether other voters are correct in their assessments, the voters may be treated as independent witnesses. If independent witnesses are equally competent and their competence is reasonable though not infallible and if we have to make up our minds one way or the other, then consulting more witnesses rather than fewer, and going by what most witnesses tell us, is always advisable. At the same time, however, the theorem is a special case of a more general principle. For any odd number of individuals i, if each i's chance of being F is p, where $.5<p<1$, and there is no dependence between the individuals as far as that property is concerned, then the

[1] Independence in the correctness of assessments follows if the following two conditions obtain:
(i) the voters are independent of other voters in what votes they cast, both given that the proposition is true and given that the proposition is false, and
(ii) each voter is equally competent in discerning the truth of the proposition when it is true as in discerning its falsity when it is false.
Condition (ii) will be made explicitly in assumption (g) in the next section. Hence, one could substitute condition (i) for assumption (c).
However, while (c) follows from (i) and (ii), the converse does not hold: (i) and (ii) need not obtain just because there is independence in the correctness of assessments.

chance that the majority of the individuals is F is higher than p and converges to 1 as the number of individuals increases. In the case of the CJT, the property F is being correct in one's assessment of a given proposition, but in general F may be any property whatsoever.

The basic model

To introduce our methodology, let us construct a function that measures the probability of the majority vote providing a correct assessment for different values of p and n. We number the voters from 1 to n. The probability that the first k voters are correct for $k = 1, \ldots, n$ is:

$$p^k \tag{1}$$

The probability that the first k voters are correct and the remaining $n-k$ voters are incorrect is:

$$p^k(1-p)^{n-k} \tag{2}$$

There are $\binom{n}{k}$ ways to pick out k individuals out of a group of n voters. Hence, the probability that precisely k out of n voters are correct is:

$$\binom{n}{k} p^k(1-p)^{n-k} \tag{3}$$

For k voters to be the majority among n voters, for an odd n, it must be the case that

$$k \text{ is an integer contained in } \left[\frac{n+1}{2}, n\right]. \tag{4}$$

So, letting M be the statement that a majority among n voters, is correct (for odd n), the probability of M being true is:

$$P(M) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k(1-p)^{n-k}. \tag{5}$$

In Fig. 1 we have plotted this function for p ranging from 0 to 1 and for $n = 3, 11, 101$. Clearly, the greater the number of voters, the more confident we may be that the majority gets it right for any particular value of p in the interval (.5, 1).

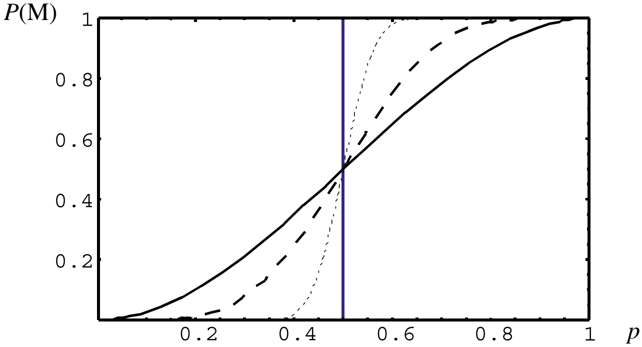

Fig. 1 The chance that the majority is correct for different levels p of voter competence and for $n = 3$ (full line), $n = 11$ (dashed line) and $n = 101$ (dotted line) voters

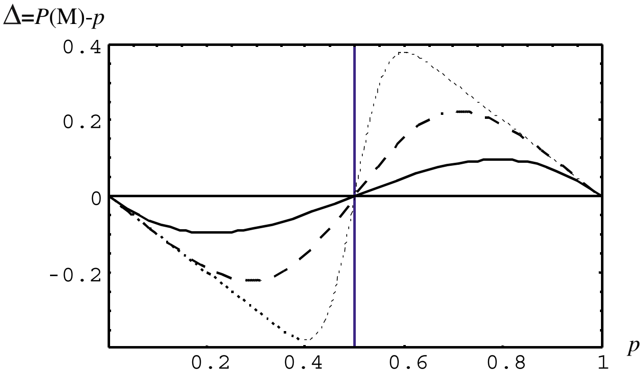

Fig. 2 The difference between the chance that the majority is correct and the chance that a single voter is correct for different levels p of voter competence and for $n = 3$ (full line), $n = 11$ (dashed line) and $n = 101$ (dotted line) voters

One might be curious to learn how much the chance that the majority vote is correct exceeds the chance that an individual voter is correct. This difference function is defined as follows:

$$\Delta = P(M) - p \tag{6}$$

and is plotted in Fig. 2. At first, when p increases above .5, the advantage of relying on the majority vote rather than on a single voter very rapidly increases, and the more so, as the number of voters increases. But then, as p comes close to 1, this majoritarian advantage decreases for the obvious reason: Since P(M) cannot be higher than 1, the difference between P(M) and p must decrease to 0.

A complex social decision

So far, we have just considered voting on a single proposition. Let us now turn to the more complex decision that is involved in the purchase of the item of equipment. Consider the following three propositions:

(S) The item of equipment meets safety standards;
(F) The item of equipment is economically feasible;
(C) The item of equipment should be purchased.

In the circumstances, the proposition C is equivalent to the conjunction S & F: The item should be purchased if and only if it meets the safety standards and is economically feasible. For each voter i in the committee that decides the issue, we assume that (i) i has a definite opinion with respect to each of the propositions S, F, and C—she either accepts it or rejects it; and that (ii) i's opinions with respect to S, F and C are internally consistent. By (i) and (ii), since i recognizes that, as things stand, C is equivalent to S&F, she accepts C if and only if she accepts each of S and F, and she rejects C otherwise.

From the previous section we adopt the assumption (a) concerning the number of voters and we also adopt the symmetry assumption (b) and the independence assumption (c) for each of the propositions S and F. Furthermore, we make two additional symmetry assumptions, (d) and (e), and four additional independence assumptions, (f), (g), (h) and (i):

(d) *The ex ante chance that an item of equipment considered by the board meets safety standards is the same as the ex ante chance that it is economically feasible*. Consequently, we let $P(\text{S})$ and $P(\text{F})$, the ex ante probabilities of S and F, to be equal to q, where $0 < q < 1$. Needless to say, this might not be true: The chances that an item meets the safety standards and is economically feasible may differ. To avoid computational complexity, we shall ignore this complication. We will distinguish between *stringent* contexts, in which fewer items of equipments meet the safety standards and are economically feasible (low q), *lenient* contexts in which more items pass the bar (high q), and *intermediate* contexts that are in between the stringent and the lenient ones.
(e) *There is an equal chance that a voter is correct in her assessment of safety standards as that she is correct in her assessment of economic feasibility*. The parameter $p \in (.5, 1)$ is the common measure of competence of each voter's assessment of S as well as of F.
(f) *Safety features and economic feasibility are independent factors*. Learning that S does not teach us anything about the chance of F nor vice versa.
(g) *Whether a voter is correct in her assessment of safety and economic feasibility is equally probable whether or not the item of equipment is in fact safe or economically feasible*. Each voter has a chance p of correctly saying of what is that it is, and an equal chance p of saying of what is not that it is not: we conceive of the reliability of a voter in the same way as of a medical test that yields the same proportion of false positives and false negatives.
(h) *Whether a voter is correct in assessing safety is independent of whether she is correct in assessing economic feasibility, and vice versa.* One might make the following objection with regard to this assumption: Learning that a particular voter is correct, say, in assessing economic feasibility may give us reason to trust her judgment, which may in turn increase our confidence in her assessment of safety. This is a reasonable objection, but we will assume a certain modularity of competence here: We conceive of assessing safety and economic feasibility as unrelated areas of expertise; one's competence in one area does not reflect on one's competence in the other.
(i) *Whether a voter is correct in assessing safety is independent of other voters being correct in assessing economic feasibility, and vice versa.* Learning about others being correct in assessing one factor does not change the chance that a particular voter is correct in her assessment of the other factor.

At certain junctions, our argument will require various conditional independences that are entailed by our independence assumptions. Using graphical models of conditional independence structures we show in Appendix 2 that these entailments hold.

Modeling the premise-based procedure

The committee members vote on S and on F separately. The item will be purchased if and only if a majority casts a positive vote on S and a majority casts a positive vote on F. We need to distinguish between four different *situations* as regards the truth of propositions S and F:

(C1) S & F

(C2) not-S & F

(C3) S & not-F

(C4) not-S and not-F.

The item ought to be purchased in situation C1, but not in situations C2 through C4. On grounds of assumptions (d) and (f), we have:

$$P(\mathrm{C1}) = q^2; \tag{7}$$
$$P(\mathrm{C2}) = P(\mathrm{C3}) = q(1-q); \tag{8}$$
$$P(\mathrm{C4}) = (1-q)^2. \tag{9}$$

Let $\mathrm{M}^{\mathrm{pbp}}$ be the proposition that the premise-based procedure will yield a correct assessment. We need to assess the chance of $\mathrm{M}^{\mathrm{pbp}}$ in each of the four situations, i.e. to determine $P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{Ci})$ for i = 1,…, 4. Consider C1. In that situation, the only way that the premise-based procedure will yield a correct assessment is when the majority is correct on S and the majority is correct on F. In Appendix 2 (Fact 1) we show that the majority being correct on S is independent of the majority being correct on F given that a particular situation Ci obtains. Hence,

$$P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C1}) = P(\mathrm{M})^2. \tag{10}$$

Consider C2. In that situation, S is false but F is true. There are three mutually exclusive ways in which the committee can reach the correct decision that the item should not be purchased: (i) the majority is right both in their assessment of S and in their assessment of F; (ii) the majority is right in their assessment of S, but wrong in their assessment of F; (iii) the majority is wrong in their assessment of S and in their assessment of F. Hence,

$$P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C2}) = P(\mathrm{M})^2 + P(\mathrm{M})(1-P(\mathrm{M})) + (1-P(\mathrm{M}))^2. \tag{11}$$

Note that in case (i) the right decision is reached for the right reasons, while in cases (ii) and (iii) the right decision is reached for the wrong reasons. C3 is analogous to C2:

$$P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C3}) = P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C2}). \tag{12}$$

Consider situation C4. S and F are both false. There are three mutually exclusive ways in which the committee can reach the correct decision that the item should not be purchased: (i) the majority is right in their assessment of S and in their assessment of F; (ii) the majority is right in their assessment of S, but wrong in their assessment of F; (iii) the majority is wrong in their assessment of S and is right in their assessment of F. Hence,

$$P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C4}) = P(\mathrm{M})^2 + 2P(\mathrm{M})(1-P(\mathrm{M})). \tag{13}$$

Again, note that in case (i) the right decision is reached for the right reasons, while in cases (ii) and (iii) the right decision is reached for the wrong reasons.[2]

The overall chance that the premise-based procedure will yield a correct assessment of whether the item ought to be purchased is:

$$P(\mathrm{M}^{\mathrm{pbp}}) = \sum_{i=1}^{4} P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{C}i)P(\mathrm{C}i) \tag{14}$$

This is the chance that the premise-based procedure yields a correct assessment *for whatever reasons*, i.e. whether or not the assessment will be made on the right grounds. But one might want to know what the chance is that this procedure will yield a correct assessment *for the*

[2] In another (weaker) sense, one might however argue that in C4 the right decision always is reached on the right grounds, even in cases (ii) and (iii). In both these cases, there is some requirement (either safety or economic feasibility) that the majority correctly deems to be violated. This right ground would by itself suffice for the committee to draw the right conclusion that the purchase should not be made. Still, on the road to the right decision, the committee commits a mistake with respect the other requirement. Thus, in the sense that we are concentrating on in this paper, the grounds for the committee's decision are partly wrong.

right reasons only. Let's refer to this chance as $P(\mathrm{M}^{\mathrm{pbp-rr}})$. In each of the four situations Ci, this chance is $P(\mathrm{M})^2$, and hence $P(\mathrm{M}^{\mathrm{pbp-rr}})$, unlike $P(\mathrm{M}^{\mathrm{pbp}})$, is independent of the situation:

$$P(\mathrm{M}^{\mathrm{pbp-rr}}) = P(\mathrm{M})^2. \tag{15}$$

Modeling the conclusion-based procedure

We turn to the conclusion-based procedure. Let $P(\mathrm{V})$ be the chance that a particular voter is correct in her vote on whether the item ought to be purchased. To determine $P(\mathrm{V})$, we will need to consider each situation. In situation C1, to cast the correct vote, the voter will need to be correct on both S and F. In Appendix 2 (Fact 2) we show that a particular voter being correct on S is independent of her being correct on F in each of the situations C1,…,C4. Following the same reasoning as for entries (10),…,(13),

$$P(\mathrm{V}|\mathrm{C1}) = p^2. \tag{16}$$

$$P(\mathrm{V}|\mathrm{C2}) = P(\mathrm{V}|\mathrm{C3}) = p^2 + p(1-p) + (1-p)^2. \tag{17}$$

$$P(\mathrm{V}|\mathrm{C4}) = p^2 + 2p(1-p). \tag{18}$$

For each situation C1,…,C4, we calculate the chance that the majority reaches the right decision on whether the item ought to be purchased. In Appendix 2 (Fact 3) we show that a particular voter being correct on whether the item ought to be purchased is independent of other voters being correct on this issue, given that a particular situation C1,…,C4 obtains. Following the same reasoning as for entries (1),…,(5), we get:

$$P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C}i) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} P(\mathrm{V}|\mathrm{C}i)^k (1 - P(\mathrm{V}|\mathrm{C}i))^{n-k} \tag{19}$$

Subsequently, we calculate the overall chance that the majority will reach a correct decision on whether the item ought to be purchased:

$$P(\mathrm{M}^{\mathrm{cbp}}) = \sum_{i=1}^{4} P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C}i)P(\mathrm{C}i) \tag{20}$$

This is the chance that the conclusion-based procedure leads to a correct assessment *for whatever reasons*, right or wrong. But once again, one might want to know what the chance is that this procedure will lead to a correct assessment *for the right reasons* only (rr), i.e. what the chance is that a majority among the voters makes the right assessment of the conclusion for the right reasons. In each of the four situations Ci, the chance that a particular voter casts a correct vote for the right reasons is p^2 and hence, following the same reasoning as for entries (1)–(5):

$$P(\mathrm{M}^{\mathrm{cbp-rr}}) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} (p^2)^k (1 - p^2)^{n-k}. \tag{21}$$

The two procedures compared with the competence of a single voter

In the last section, we calculated the competence of a single voter to reach the correct complex decision in each situation, i.e. $P(\mathrm{V}|\mathrm{C}i)$ for i = 1,…,4. This allows us to calculate the overall competence of a single voter:

$$P(\mathrm{V}) = \sum_{i=1}^{4} P(\mathrm{V}|\mathrm{C}i)P(\mathrm{C}i). \tag{22}$$

In this section we will consider whether the probabilities that the majority is correct in, respectively, the premise-based and the conclusion-based procedures exceed the probability that a single voter is correct. We let n range over odd numbers from 3 through infinity and p over the open set (.5, 1).

We start with the premise-based procedure. It is not surprising that the probability that the majority is correct on this procedure always exceeds the probability that a single voter is correct. To show this we calculate $P(\mathrm{M}^{\mathrm{pbp}})$, using entries (7) through (14), and we calculate $P(\mathrm{V})$, using entries (7) through (9) and (16) through (18) and (22). Subsequently we calculate the difference $P(\mathrm{M}^{\mathrm{pbp}}) - P(\mathrm{V})$. We first consider the special case of $q = .5$: Given assumptions (d) and (f), this is a case in which $P(\mathrm{S\&F}) = (.5)^2$, i.e. one out of four items is worth purchasing. Some algebraic manipulation yields:

$$P(\mathrm{M}^{\mathrm{pbp}}) - P(\mathrm{V}). = 1/2(P(\mathrm{M})^2 + 1) - 1/2(p^2 + 1) = 1/2(P(\mathrm{M})^2 - p^2), \tag{23}$$

which is larger than 0, since $P(\mathrm{M}) > p$ for the relevant values of p and n. In the general case, some algebraic manipulation yields:

$$\begin{aligned} P(\mathrm{M}^{\mathrm{pbp}}) - P(\mathrm{V}) = {} & (P(\mathrm{M}) - p)((2 - P(\mathrm{M}) - p)(1 - q)^2 + q^2 \\ & + (P(\mathrm{M}) + p - 1)(1 - (1 - q)^2), \end{aligned} \tag{24}$$

which again is larger than 0, since $P(\mathrm{M}) > p$ and $0 > q > 1$ for the relevant values of p and n.[3] Hence, we can conclude in general that the premise-based procedure is better at tracking truth for whatever reasons in complex social decisions than a single voter.

We now turn to the conclusion-based procedure. Here the results are more surprising: As we shall see, in lenient contexts (high q), the probability that the majority is correct on this procedure might be lower than the probability that a single voter is correct. But let us consider the intermediary context first. In Fig. 3, we plot $P(\mathrm{M}^{\mathrm{cbp}})$ for $n = 51$ and $n = 501$ along with $P(\mathrm{V})$, for $q = .5$. As n converges to infinity, the function $P(\mathrm{M}^{cbp})$ converges to a step function: In the limit, (i) for all $p \in (0, 1 - \sqrt{.5})$, $P(\mathrm{M}^{\mathrm{cbp}})$ tends to $2q(1 - q) = .5$, (ii) for all $p \in (1 - \sqrt{.5}, \sqrt{.5})$, $P(\mathrm{M}^{\mathrm{cbp}})$ tends to $2q(1 - q) + (1 - q)^2 = .75$, and (iii) for all $p \in (\sqrt{.5}, 1)$, $P(\mathrm{M}^{\mathrm{cbp}})$ tends to 1. Let us explain these results for each step:

Step 1: $p \in (0, 1 - \sqrt{.5})$. If we plug in values for p in this range in (16)–(18), then we notice that $P(\mathrm{V}|\mathrm{C2})$ and $P(\mathrm{V}|\mathrm{C3})$ exceed .5, but $P(\mathrm{V}|\mathrm{C1})$ and $P(\mathrm{V}|\mathrm{C4})$ fall below .5. Hence, by the CJT, $P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C2})$ and $P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C3})$ converge to 1, but $P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C1})$ and $P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{C4})$ converge to 0 as n approaches infinity. In the limit, therefore, by (8) and (20), $P(\mathrm{M}^{\mathrm{cbp}}) = 1 \times P(\mathrm{C2}) + 1 \times P(\mathrm{C3}) = 2q(1 - q)$.

Step 2: $p \in (1 - \sqrt{.5}, \sqrt{.5})$. We follow the same reasoning as in step 1. For the values of p in this range, $P(\mathrm{V}|\mathrm{C2})$ through $P(\mathrm{V}|\mathrm{C4})$ exceed .5, but $P(\mathrm{V}|\mathrm{C1})$ falls below .5. Hence, by the CJT, (8), (9) and (20), $P(\mathrm{M}^{\mathrm{cbp}}) = 1 \times P(\mathrm{C2}) + 1 \times P(\mathrm{C3}) + 1 \times P(\mathrm{C4}) = 2q(1 - q) + (1 - q)^2 = 1 - q^2$.

Step 3: $p \in (\sqrt{.5}, 1)$: Again, we follow the same reasoning as in step 1. For the values of p in this range, $P(\mathrm{V}|\mathrm{C1})$ through $P(\mathrm{V}|\mathrm{C4})$ exceed .5. Hence, by the CJT and (20), $P(\mathrm{M}^{\mathrm{cbp}}) = 1$.

[3] We owe this expression of $P(\mathrm{M}^{\mathrm{pbp}}) - P(\mathrm{V})$ in (24) to Stephan Hartmann.

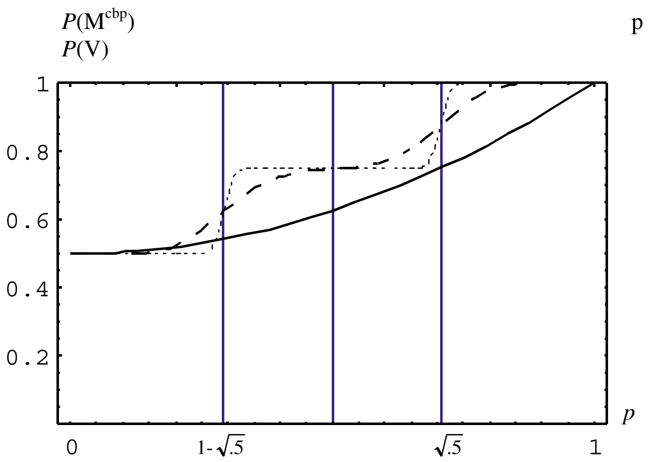

Fig. 3 The chance that the majority vote is correct on the conclusion-based procedure for 51 voters (dashed line) and for 501 voters (dotted line), along with the chance that a single voter is correct (full line), for $q = .5$

For lower values of n, e.g. for $n = 51$, the curve for the $P(\mathrm{M}^{\mathrm{cbp}})$ is increasing in two waves. As the value of n rises, e.g. for $n = 501$, the curve flattens out into three horizontal plateaus, which continue to broaden, converging to the critical values as n approaches infinity.

How does $P(\mathrm{M}^{\mathrm{cbp}})$ compare to $P(\mathrm{V})$ for $q = .5$? From (16) through (18) and (22), it follows that $P(\mathrm{V}) = 1/2(p^2 + 1) = .75$ for $p = \sqrt{.5}$. But we have just learned that $P(\mathrm{M}^{\mathrm{cbp}})$ converges to .75 as p approaches $\sqrt{.5}$ from below. Hence, $P(\mathrm{M}^{\mathrm{cbp}})$ converges to $P(\mathrm{V})$ for $p = \sqrt{.5}$ as n approaches infinity. This is a curious result: As the number of voters approaches infinity, the chance that the majority of the voters is correct tends to the same value as the chance that a single voter is correct, provided that the competence p of each voter is $\sqrt{.5}$.

The results are even more surprising when we relax the assumption that $q = .5$. As n approaches infinity, the central plateau for $P(\mathrm{M}^{\mathrm{cbp}})$, i.e. the horizontal curve for that function over the p-interval $(1 - \sqrt{.5}, \sqrt{.5})$, has the height $1 - \mathrm{q}^2$. $P(\mathrm{V})$ is an increasing function of p which takes on the value of $2\mathrm{q}(1-\mathrm{q})$ for $p = 0$, and 1 for $p = 1$. As we increase the value of q to, say, $q = .65$, then the central plateau of $P(\mathrm{M}^{\mathrm{cbp}})$ moves down and drops below the curve for $P(\mathrm{V})$ for certain values of p greater than .5 but smaller than $\sqrt{.5}$ (Fig. 4). That is, there are certain levels of competence p for which the vote of a single voter on a complex decision is more likely to be correct than the majority vote following the conclusion-based procedure. This phenomenon occurs (i) for higher values of q, i.e. for more lenient contexts, (ii) for relatively low $p > .5$, but not necessarily for those values of p that are only slightly over .5, as we can see in the Fig. 4, and (iii) for higher n, since the central plateau of $P(\mathrm{M}^{\mathrm{cbc}})$ flattens out and then broadens as the value of n increases.

If our objective is truth for whatever reasons, how do the two procedures compare to each other?

We already know that the premise-based procedure will perform better for certain values of n, p and q, since $P(\mathrm{M}^{\mathrm{pbp}}) > P(\mathrm{V})$ for all values of these parameters and $P(\mathrm{V}) > P(\mathrm{M}^{\mathrm{cbp}})$ for some values of these parameters. But are there any values of n, p and q for which $P(\mathrm{M}^{\mathrm{cbp}}) > P(\mathrm{M}^{\mathrm{pbp}})$? Let us set q once again at .5. We notice that $P(\mathrm{M}^{\mathrm{cbp}})$ starts rising

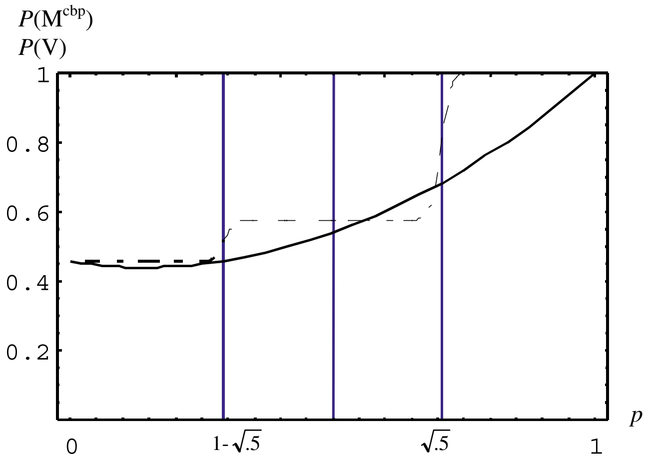

Fig. 4 The chance that the majority vote is correct on the conclusion-based procedure for 501 voters (dashed line) and the chance that a single voter is correct on a complex decision (full line), for $q = .65$

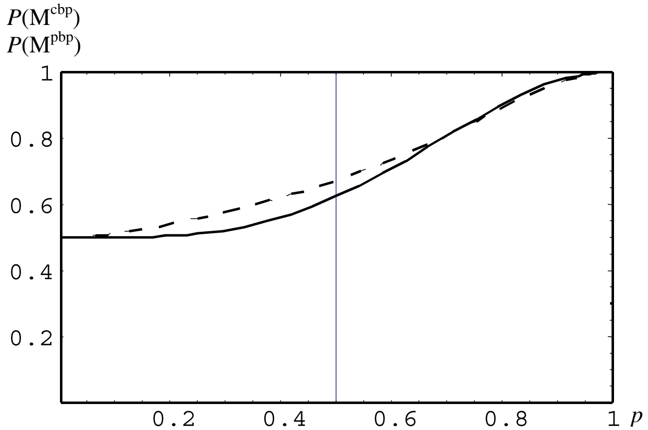

Fig. 5 The chance that the majority vote is correct on the conclusion-based procedure (dashed line) and on the premise-based procedure (full line), for three voters and $q = .5$

for lower values of p than $P(\mathrm{M}^{\mathrm{pbp}})$, but the slope of this rise decreases sooner for $n = 3$ in Fig. 5 or it reaches a plateau for $n = 101$ in Fig. 6. Thus, $P(\mathrm{M}^{\mathrm{cbp}})$ is higher than $P(\mathrm{M}^{\mathrm{pbp}})$ only for lower values of p. Furthermore, the range of values of p for which $P(\mathrm{M}^{\mathrm{cbp}})$ exceeds $P(\mathrm{M}^{\mathrm{pbp}})$ is broader for a lower value of n in Fig. 5 than for a higher value of n in Fig. 6.

Finally, we know from the previous section that we can raise the platform of $P(\mathrm{M}^{\mathrm{cbp}})$ by lowering the value of q: As we pull up the platform, the range of values of p for which $P(\mathrm{M}^{\mathrm{cbp}})$ exceeds $P(\mathrm{M}^{\mathrm{pbp}})$ will broaden. In Fig. 7 we have plotted curves that indicate for what values of q and p the conclusion-based procedure does better than the premise-based procedure for $n = 3$, 11, 101.

This graph confirms our comparisons of the behavior of the functions $P(\mathrm{M}^{\mathrm{cbc}})$ and $P(\mathrm{M}^{\mathrm{pbp}})$ for different values of p, q and n: The conclusion-based procedure tends to do

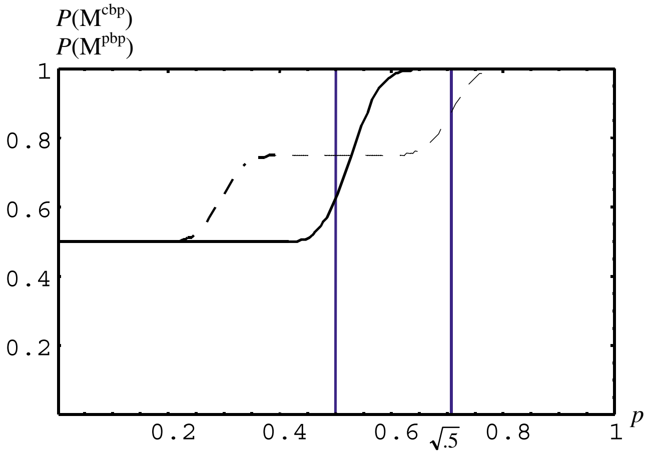

Fig. 6 The chance that the majority vote is correct on the conclusion-based procedure (dashed line) and on the premise-based procedure (full line), for 101 voters and $q = .5$

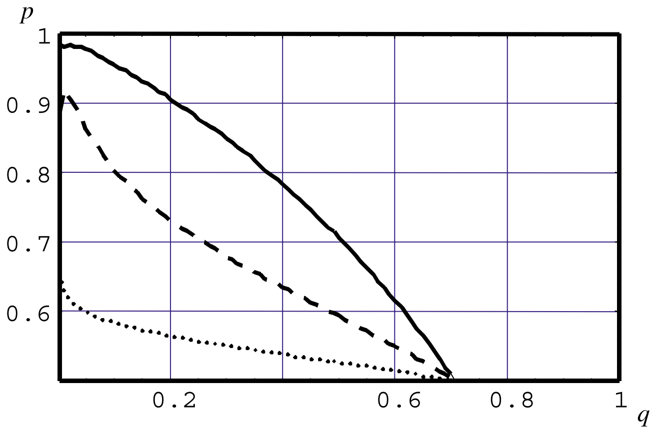

Fig. 7 Phase diagram for $n = 3$ (full line), 11 (dashed line), 101 (dotted line), for p between .5 and 1, and for q between 0 and 1.Underneath the curves, the conclusion-based procedure is the better truth-tracker; above the curves, the premise-based procedure is the better truth-tracker

better as we decrease the size of the committee, the competence of the voters and the leniency of the context.

The following reasons account for these facts: (i) The conclusion-based procedure does better than the premise-based procedure when it comes to reaching truth by mistake, i.e. for the *wrong* reasons. Less competent voters are more prone to commit mistakes, and the room for truth by mistake is larger in less lenient contexts, in which the probability of the conjunction being false is higher. As we know, the truth about a conjunction can be reached by mistake only when the conjunction is false. (ii) As we shall see in the next section, as n increases, with less competent voters the premise-based procedure does increasingly better than the conclusion-based procedure, when it comes to reaching truth for the *right* reasons. But for small numbers of voters, this advantage of the premise-based procedure is smaller and thus it cannot outweigh its disadvantage as regards reaching truth by mistake.

If our objective is truth for the right reasons, how do the two procedures compare to each other?

Concerning the capacity of the two procedures as regards truth-tracking *for the right reasons*, it is easy to see that the premise-based procedure is superior in that respect. This procedure yields the correct assessment based on the right reasons whenever (i) there is a majority that correctly assesses one premise and also (ii) a majority that correctly assesses the other premise. The conclusion-based procedure, on the other hand, correctly assesses the conclusion for the right reasons if and only if (iii) there is a majority that correctly assesses *both* premises. Obviously, (iii) entails (i) and (ii), but not vice versa. Therefore, whenever the conclusion-based procedure makes the right assessment for the right reasons, the premise-based procedure would do so as well. At the same time, there are possible cases in which the premise-based procedure would make a right assessment for the right reasons but the conclusion-based procedure would fail in that respect. Such cases (in which (i) and (ii) hold, but (iii) does not) have non-zero probability as long as the voters' competence with respect to one premise is at least partly independent of their competence with respect the other premise. And we have assumed that these competences are fully independent for each other.

Still, it is one thing to know that premise-based procedure is superior as a truth-tracker for the right reasons, but yet another to determine the extent of this superiority. We now proceed to this task. We calculated the $P(\mathrm{M}^{\text{pbp-rr}})$ in (15) and $P(\mathrm{M}^{\text{cbp-rr}})$ in (21). In Fig. 8 we plot both $P(\mathrm{M}^{\text{pbp-rr}})$ and $P(\mathrm{M}^{\text{cbp-rr}})$ and in Fig. 9 we plot their difference. $P(\mathrm{M}^{\text{pbp-rr}})$ always exceeds $P(\mathrm{M}^{\text{cbp-rr}})$, but the difference is particularly large for values in the range $(.5, \sqrt{.5})$, and the more so for greater numbers of voters. It is easy to see why this is so. By the CJT, $P(\mathrm{M})$ tends to 0 for $p < .5$ and to 1 for $p > .5$ as the value of n approaches infinity, while $P(\mathrm{M}) = .5$ for $p = .5$. Hence, $P(\mathrm{M}^{\text{pbp-rr}}) = P(\mathrm{M})^2$ will tend to 0 for $p < .5$ and to 1 for $p > .5$, while it equals .25 for $p = .5$. On the other hand, by the CJT, the right-hand side of equation (21) will tend to 0 for $p^2 < .5$ and to 1 for $p^2 > .5$ as the value of n approaches infinity, while it will equal .5 for $P^2 = .5$. Hence, $P(\mathrm{M}^{\text{cbp-rr}})$ will tend to 0 for $P < \sqrt{.5}$ and to 1 for $p > \sqrt{.5}$ as the value of n approaches infinity, while $P(\mathrm{M}^{\text{cbp-rr}}) = .5$ for $p = \sqrt{.5}$. We can conclude that as n approaches infinity, $P(\mathrm{M}^{\text{pbp-rr}}) = p(\mathrm{M}^{\text{cbp-rr}}) = 0$ for $p < .5$, $P(\mathrm{M}^{\text{pbp-rr}}) = 1$ and $P(\mathrm{M}^{\text{cbp-rr}}) = 0$ for $p \in (.5, \sqrt{.5})$ and $P(\mathrm{M}^{\text{pbp-rr}}) = P(\mathrm{M}^{\text{cbp-rr}}) = 1$ for $p > \sqrt{.5}$. For lower values of n, the curves are less steep and, as a consequence, the

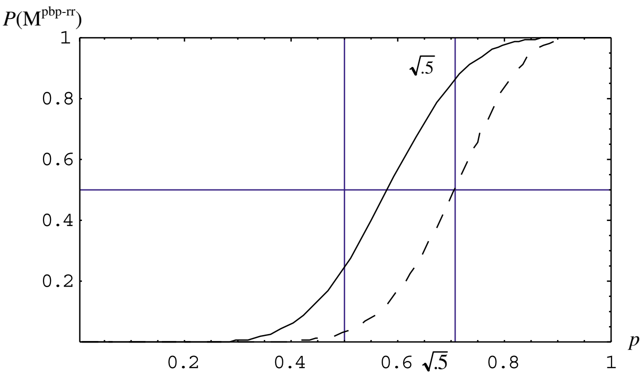

Fig. 8 The chance that the majority vote is correct for the right reasons, for the premise-based procedure (full line) and for the conclusion-based procedure (dashed line), for 11 voters

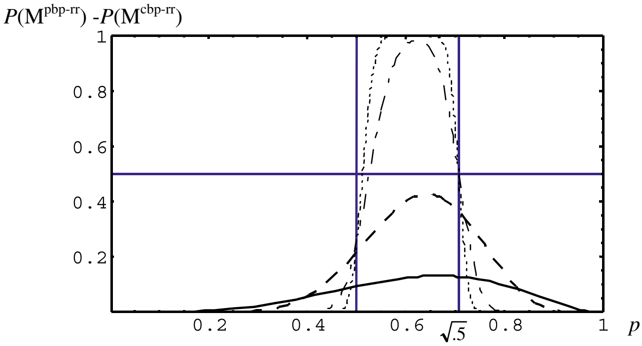

Fig. 9 The difference between the chances that the majority vote is correct for the right reasons for the premise-based procedure and for the conclusion-based procedure, for $n = 3$ (full line), $n = 11$ (dashed line), $n = 101$ (dot-dashed line) and $n = 501$ (dotted line)

differences between $P(\text{M}^{\text{pbp-rr}})$ and $P(\text{M}^{\text{cbp-rr}})$ are smaller for $p \in (.5, \sqrt{.5})$, but there is a broader range of values of p which $P(\text{M}^{\text{pbp-rr}})$ exceeds $P(\text{M}^{\text{cbp-rr}})$.

So if what we are after is truth for the right reasons, the truth-tracking potential of the premise-based procedure is never worse than that of the conclusion-based procedure. It tends to be substantially better for voters whose assessments are slightly better than random. For small committees, the difference is small, but the range of competence levels for which the premise-based procedure exceeds the conclusion-based procedure is wide. For larger committees, the difference is larger but the range of competence levels for which the premise-based procedure exceeds the conclusion-based procedures converges to the p-interval $(.5, \sqrt{.5})$.

Discussion

Independence

Our model is constructed under a series of idealizations of independence. These idealizations may be more or less realistic in particular empirical situations. It is an open question how robust our results are if we relax various combinations of idealizations, by substituting positive or negative relevance for independence, as the case may be. This exercise needs to be undertaken relative to a particular empirical situation.

An interesting application of our model is the tenure vote in north-American institutions of higher learning. In many institutions, both the teaching and the research skills of the candidates are deemed to be relevant to the decision: The candidates are required to meet certain standards on both scores. The dean's decision is informed by a faculty vote in the home department of the candidate. The dean might ask each faculty member to assess the candidate on teaching and research and cast a yes vote for tenure if and only if she deems the candidate to be worthy on both teaching and research. Tenure will be granted just in case there is majority support for the candidate. Or the dean might ask each faculty member to cast a vote on whether the candidate is worthy of teaching and to cast a vote on whether the candidate is worthy on research. Tenure will be granted just in case there is majority support on teaching and there is majority support on research. This decision problem has the same structure as the decision concerning the purchase of a new item of equipment. One might therefore be tempted to conclude that the conclusion-based procedure is a better truth-tracker than the premise-based procedure as fewer candidates meet the mark on teaching and research in a particular school, as the faculty members are less qualified in their assessments and as there are fewer voters in the department.

However, the independence assumptions are somewhat less plausible in this case than in our example. For instance, one might object to independence assumption (f): Typically there is some connection between the candidate's research and teaching skills. One might argue that there is positive relevance, since teaching typically is enhanced by research and vice versa. But one might equally argue that there is negative relevance, since time devoted to research is time taken away from teaching, and vice versa. Or one might object to independence assumption (h): Typically there is a connection between the assessment skills of senior faculty members on teaching and on research. One might argue that there is positive relevance, since both assessments will be influenced by the degree of bias, the level of expertise and the care invested in the evaluation process. On the other hand, one might argue for negative relevance: faculty members typically profile themselves either as good teachers or as good researchers, and this dichotomy might carry over in the quality of their assessment of tenure candidates.

We mentioned earlier that the CJT still stands even if assumption (c) is to some extent violated, e.g. when opinion leaders have a limited influence. As an analytical exercise, we relaxed the assumption of independence (f) in the direction of positive relevance and in the direction of negative relevance. (Results are omitted.) As it turned out, our general results proved to be quite robust under these relaxations. Of course, the idiosyncrasies of every situation would need to be examined and modeled accordingly, but there is at least some reason to believe that our general results are not an artifact of unrealistic independence assumptions.

The cost of error

We have shown under what conditions the conclusion-based procedure is a better truth-tracker than the premise-based procedure. The conclusion-based approach gains its advantage because it is better at reaching truth for the wrong reasons. Truth for the wrong reasons can only be reached when the conjunction is false. Hence, the conclusion-based account gains its advantage from providing a more accurate assessment of non-acceptable items of equipment or of non-acceptable candidates for tenure. Now, a company or an institution of higher learning may be more concerned about the danger that non-acceptable items or unqualified candidates would be incorrectly accepted rather than about the opposite danger that acceptable items or qualified candidates would be incorrectly rejected. Hence, they will be somewhat more favorable to the conclusion-based account. Under other circumstances (when there is a shortage of available alternatives), the preferences of the committee may be different: the danger of incorrectly rejecting an acceptable alternative may be deemed to be more serious than the opposite mistake. Our model rests on the assumption that one cares about truth *per se* and does not prefer one kind of error to the other. If the cost of incorrect acceptance differs from the cost of incorrect rejection, then the balance in the evaluation of the premise-based procedure and the conclusion-based procedure will shift. In order to give a precise answer as to whether we should favor the premise-based procedure or the conclusion procedure when the types of error committed carry different costs, we would need to introduce utility values into the model and develop a more complex decision-theoretic approach.

Deliberative democracy and reliabilism

In the Section 'Two voting procedures', we have seen that the premise-based procedure is much more congenial to the ideal of deliberative democracy. As we have shown in the Section 'If our objective is truth for whatever reasons, how do the two procedures compare to each other?', however, in some types of cases (with fewer voters, lower competence levels, and within more stringent contexts) the conclusion-based procedure is a more reliable truth-tracker. Should this observation be worrisome for the adherents of deliberative democracy? Pettit (2001) denies this. What is decisive for the deliberative democrat is a procedure's potential when it comes to reaching truth *for right reasons* and in this respect the premise-based procedure is definitely superior. As Pettit puts it:

> When a person or community makes a correct judgment that A for the wrong deliberative reasons, then we deny that they understand why it is the case that A, or that they know that A. But the ideal of deliberative democracy, as that has been articulated on all sides, is closely bound to the alleged prospect of an increase of understanding and perhaps knowledge on the part of individuals in the community, and the group as a whole; there is no suggestion that it merely increases the likelihood of serendipitous error. The ideal supposes that in relying on deliberation to guide them towards a collective judgment, people will be guided by right reasons: that is, by reasons that are sound as well as supportive. (Pettit, 2001; quoted with small changes in the notation)

In other words, what deliberative democrats are after is not simply true beliefs but increased understanding and knowledge, i.e. true beliefs for the right reasons. This applies, in particular, to the beliefs of the group as the whole. Obviously, the premise-based procedure is especially helpful in this respect. However, leaving the issue of understanding aside, we should point out that Pettit's argument rests on a contested concept of knowledge. The classical analysis of knowledge as true belief based on right reasons does not appeal to epistemologists of externalist persuasion. An especially popular externalist view takes knowledge to be a true belief arrived at by *reliable methods* (e.g. Goldman, 1979). Thus, it is the truth-tracking potential of the methods used that determines the epistemic status of the true belief under consideration. The epistemic status of a belief increases with the reliability of the methods by which it has been reached. On this externalist view then, the conclusion-based procedure would confer a higher epistemic status to the assessments of the group in all those types of cases in which that procedure is a better truth-tracker.[4] Consequently, the superiority of that method in certain types of cases should be worrisome for deliberative democrats of an externalist bent. There is a tension between the epistemological commitment to externalism and the political commitment to deliberative democracy. In certain cases, the former commitment pulls one in the direction of the conclusion-based procedure, while the latter commitment always pulls one in the direction of the premise-based procedure.

Other inference patterns and the problem of instability

In this paper, we have only considered a very simple type of case, one in which a group takes a stand on a conjunction on the basis of the voters' views concerning the conjuncts. What can be said about complex social decisions in which premises lead to the conclusion via inference rules other than conjunction introduction? Being right in the assessment of the truth value of a proposition is equivalent to being right in the assessment of the truth value of its negation.

[4] We owe this observation to Peter Pagin.

Since the negation of a conjunction is equivalent to the disjunction of its negated conjuncts, our results concerning conjunction introduction extend to disjunction introduction just as well: We can apply them mutatis mutandis to those social decisions in which the conclusion is a disjunction that is being accepted if at least one of the disjuncts gets a majority of votes (the premise-based procedure) or if the voters who accept one or the other of the disjuncts are in the majority (the conclusion-based procedure). But what about other inference rules, such as modus ponens, disjunctive syllogism, etc.?

And what about the inference patterns in which the conclusion could be reached by a series of steps rather than in just one move? It is important to note that, in such cases, the premise-based procedure may be applied in different ways. We could either let the voters vote on the propositions from which the main conclusion immediately follows, or instead ask them to vote on the premises that lie farther back in the inference chain. As it turns out, the premise-based procedure may deliver different results depending on the level at which it is applied. In other words, unlike the conclusion-based procedure, the premise-based procedure is *unstable*, as shown by the following example:

Suppose that, in a given group, there are majorities *against* each of the propositions A, B, C, D, but also (ii) majorities *for* A∨B and *for* C∨D. Suppose the group needs to take a stand with respect to the following proposition:

$$\text{(X) } (\text{A} \vee \text{B}) \ \&\ (\text{C} \vee \text{D}).$$

If the premise-based procedure is applied at level (i), i.e. if the vote is conducted on each of the propositions A, B, C and D, then the group will first reject each of the conjuncts in X and then move on to reject X itself. But if that procedure instead is applied at level (ii), i.e. if the vote is conducted only on A∨B and on C∨D, then the group will accept X. How serious is this instability problem?

Second thoughts about independence

On second thought, it appears that some of our independence assumptions cast doubts on the epistemic reasonableness of the conclusion-based procedure.[5] In particular, according to assumption (h) in the Section 'A complex social decision' above, the competence of a voter in assessing one factor, S, is independent of her competence in assessing the other factor, F. If that assumption holds, then—from the epistemic point of view—each voter may be thought of as a coincidental combination of two 'detection instruments', s and f, for detecting factors S and F, respectively. These instruments just happen to be conjoined with each other in one voter. Another voter happens to be composed of another such arbitrarily conjoined pair of detection instruments, say, s' and f'. Now, the conclusion-based procedure lets each voter deliver a verdict on the conjunction of factors, $S\&F$. This means that such a procedure attaches a special importance to the particular way in which detection instruments happen to be conjoined in different voters. The instrument pairs (s, f) and (s', f') get to vote, but the pairs (s, f') and (s', f) don't, since they are not instanced in a single voter. Since the actual conjoining of detecting instruments is arbitrary given independence, this feature of the conclusion-based procedure is problematic and casts serious doubts on its epistemic justification. Note that the premise-based procedure avoids this objection. Unlike the

[5] We are indebted for this point to Magnus Jiborn.

conclusion-based procedure, it does not depend on the way in which detection skills happen to be combined in particular voters.[6]

These are only some of the issues that merit further research.[7]

Appendix 1

For k voters to be the majority of n even-numbered voters, it must be the case that k is an integer contained in $\left[\frac{1}{2}n+1,\ n\right]$. Following the methodology outlined in the Section 'The basic model', the probability that a majority among n even-numbered voters is correct is:

$$p(\mathrm{M}_{\mathrm{even}}) = \sum_{k=\frac{1}{2}n+1}^{n} \binom{n}{k} p^k (1-p)^{n-k}.$$

We calculate the difference function, $\Delta_{\mathrm{even}} = P(\mathrm{M}_{\mathrm{even}}) - p$, for $n = 4, 6, 8, \ldots$ In Fig. 10, we plot the difference function for $n = 4,\ n = 12$ and $n = 102$: The probability that the majority is correct exceeds the probability that a single voter is correct if and only if the competence of each voter exceeds $p(4) \approx .77$, $p(12) \approx .56$, $p(102) \approx .5056$. We have plotted the critical values $p(n)$ as a function of n in Fig. 11. Notice that the values of $p(n)$ tend towards .50 as n grows larger: The more even-numbered voters there are, the less competence is required from each voters for the probability that the majority is correct to exceed the probability that an individual voter is correct, subject to the constraint that $p > .5$.

Appendix 2

We will show by means of graphical models of conditional independence structures (see e.g. Pearl, 1988: 77–141) that the three conditional independences in Facts 1, 2 and 3 below hold. The arrows in a Directed Acyclical Graph (DAG) represent direct influences between variables, with conditional independence being understood in terms of the *shielding off* condition: Parents shield off their children from all their non-descendants in the graph. Or, in other words, the DAG respects the Parental Markov Condition:

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[6] In response to this epistemic objection to the conclusion-based procedure, one could appeal to the separateness of persons in Rawls' style, or to considerations of autonomy. Just like we are not receptacles of utility, we are not just receptacles of opinions generated by detection machines. Even if there is independence between the reliability of my opinions regarding different factors, there is still good reason to do conclusion-based voting, because these are all *my* opinions. When it comes to finding out the truth, (s, f') and (s', f) are just as good as (s, f) and (s', f'). But there is something to the intuition that every person should have a chance to express her own overall opinion about the matter.

[7] This paper was originally presented at the Entretiens d' Institut International de Philosophie in Helsinki and Tarttu, in August 2001. We wish to thank the organizers, Jaakko Hintikka and Matti Sintonen, and the participants of that meeting. We are indebted to Stephan Hartmann, Magnus Jiborn, Mats Johansson, Christian List, Peter Pagin, Philip Pettit, Thomas Schmidt, Alois Stutzer and Josh Snyder, for helpful comments and suggestions. Luc Bovens' research was supported by grants of the National Science Foundation (Science and Technology Studies—SES 00-80580), of the Graduate Committee on the Arts and Humanities of the University of Colorado at Boulder, and of the Alexander von Humboldt Foundation, the Federal Ministry of Education and Research, and the Program for Investment in the Future (ZIP) of the German Government. Wlodek Rabinowicz' research was supported by the Bank of Sweden Tercentenary Foundation.

A revised version of the paper was presented at a conference on Social Software in the Carlsberg Academy in Copenhagen, in May 2004. We are indebted to the organizers of the conference, Vincent Hendricks and Stig Andur Pedersen, and to the participants.

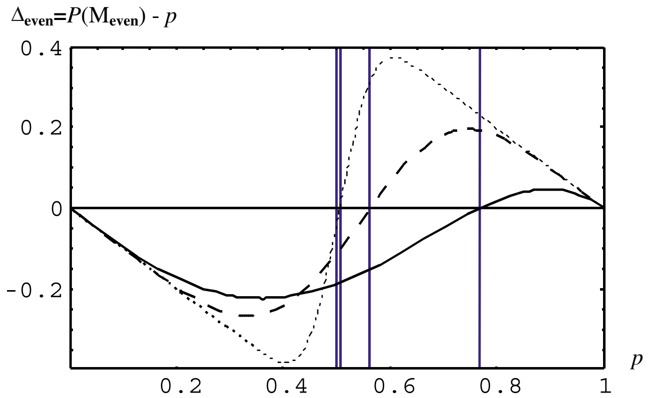

Fig. 10 The difference between the chances that the majority judgment is correct and that a single voter is correct, for $n = 4$ (full line), $n = 12$ (dashed line) and $n = 102$ (dotted line) voters. For $n = 4$, $\Delta_{\text{even}} < 0$ for $p < .77$; for $n = 12$, $\Delta_{\text{even}} < 0$ for $p < .56$; and for $n = 102$, $\Delta_{\text{even}} < 0$ for $p < .5056$

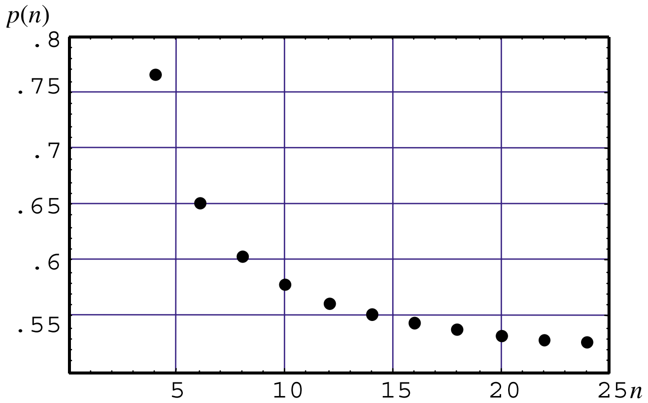

Fig. 11 $p(n)$ is the value of p beyond which the probability that the majority is correct exceeds the probability that an individual voter is correct for $n = 4, 6, \ldots, 24$

(PMC) A variable represented by a node in a Bayesian Network is independent of the variables represented by its non-descendant nodes in the Bayesian Network, conditional on all variables represented by its parent nodes.

We define the following variables:

S states whether the item of equipment is safe;
F states whether the item of equipment is economically feasible;
C states what combination of the values of S and F obtains;
Fi states whether voter i is correct about F;
Si states whether voter i is correct about S;
Vi states whether voter i is correct about the conjunction of S and F (i.e. about C);
M^F states whether the majority is correct about F;
M^S states whether the majority is correct about S;
D states whether the committee's decision to either purchase or not purchase the item of equipment (i.e. its assessment of C) is correct.

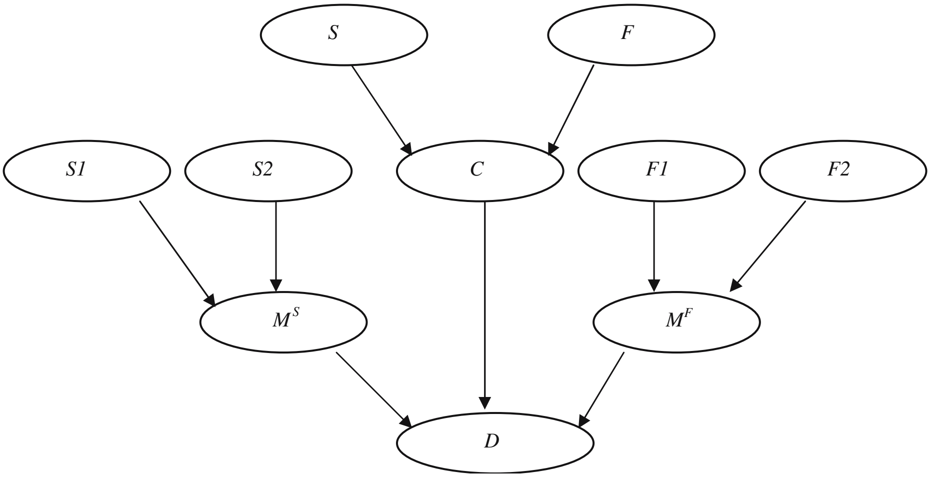

Fig. 12 Graph for the premise-based procedure

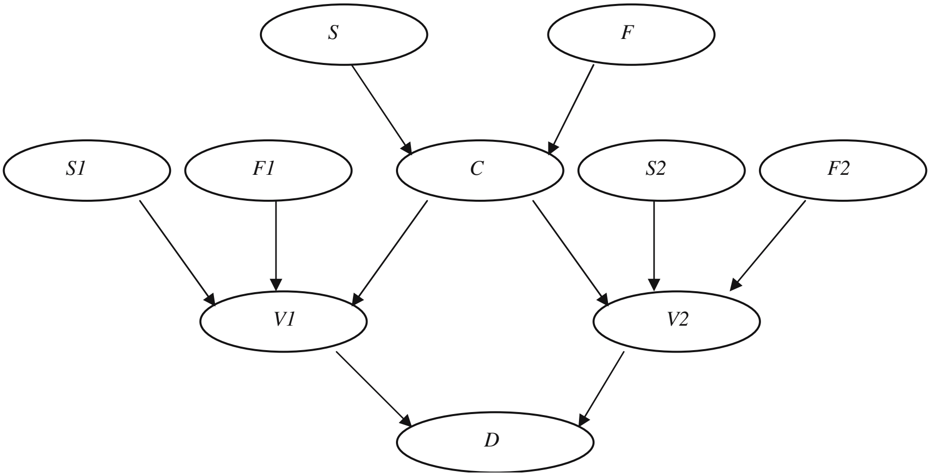

Fig. 13 Graph for the conclusion-based procedure

We have presented the DAGs for $n = 2$ voters in Fig. 12 for the premise-based procedure and in Fig. 13 for the conclusion-based procedure. It is clear how these arrows track direct influences in both procedures, except maybe for the arrows leaving the nodes with the variable C. We explain: Note that for the premise-based procedure, if S & F obtains, it is much more difficult for the committee to draw the correct conclusion about whether the conjunction of S and F obtains, than if S & not-F, not-S & F or not-S & not-F obtain. If S & F obtains, then the committee needs to be correct about both S and F to be correct about the conjunction, but if any of the other combinations obtains, being incorrect about some combinations of S and F will also get the committee to the correct truth-value for the conjunction. The same argument holds *mutatis mutandis* for the conclusion-based procedure. For individual voters to be correct about the conjunction is much more difficult if both conjuncts are true. For any sets of variables α, β, and γ, let $\alpha\_||\_\beta$ and $\alpha\_||\_\beta|\gamma$ stand, respectively, for the claims that α is independent of β and that α is independent of β given γ. Since the parents of S, F, Si and Fi are the empty set in both graphs, we can read off the following independences in both graphs (or, more precisely, in the generalizations of these graphs for n voters) by means of the (PMC):[8]

$$S\_||\_F$$

$$Si\_||\_S1, \ldots, Si-1, Si+1, \ldots, Sn, S, F, Fi, F1, \ldots, Fi-1, Fi+1, \ldots, Fn, \quad \text{for } i = 1, \ldots, n$$

$$Fi\_||\_F1, \ldots, Fi-1, Fi+1, \ldots, Fn, S, F, Si, S1, \ldots, Si-1, Si+1, \ldots, Sn, \quad \text{for } i = 1, \ldots, n$$

The first independence corresponds to assumption (f) and the latter two independences correspond to assumptions (c), (g), (h) and (i). The standard method to check whether a particular conditional independence holds in the graphical model is to appeal to the d-separation criterion.[9] By means of this criterion, we can read off the following conditional independences in the graph in Fig. 12:

$$\text{Fact 1}: M^S\_||\_M^F|C$$

and in the graph in Fig. 13:

$$\text{Fact 2}: Si\_||\_Fi|C \quad \text{for } i = 1, \ldots, n.$$

$$\text{Fact 3}: Vi\_||\_V1, \ldots, Vi-1, Vi+1, \ldots, Vn|C \quad \text{for } i = 1, \ldots, n.$$

References

Borland, P.J. (1989). Majority systems and the Condorcet Jury Theorem. *Statistician, 38*, 181–189.

Brennan, G. (2001). Collective coherence? *International Review of Law and Economics, 21*, 197–211.

Chapman, B. (1998a). Law, incommensurability, and conceptually sequenced argument. *University of Pennsylvania Law Review, 146*, 1487–1582.

Chapman, B. (1998b). More easily done than said: Rules, reason and rational Social choice. *Oxford Journal of Legal Studies, 18*, 293–329.

Cohen, J. (1986). An epistemic conception of democracy. *Ethics, 97*, 26–38.

Condorcet, J.A. de (1785). *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*, l'Imprimerie Royale, Paris, partly translated into English in McLean and Urken (1995).

Elster, J. (Ed.) (1998). *Deliberative democracy*. Cambridge University Press.

Estlund, D. (1990). Democracy without preference. *Philosophical review, 49*, 397–424.

Estlund, D. (1993). Making truth safe for democracy. In: David Copp, Jean Hampton and John E. Roemer, *The Idea of Democracy,* (Eds.) New York: Cambridge University Press, 71–100.

[8] More accurately, by means of the (PMC) and the semi-graphoid axiom of decomposition, which states that for any sets of variables α, β, γ, δ, if α is independent of β and γ given δ, then α is independent of β given δ and α is independent of γ given δ.

[9] For any sets of variables α, β, γ, to determine whether α is independent of a β given γ, construct a list that specifies all the possible pairs containing a variable in α and a variable in β. Focus on some entry (A, B) with $A \in \alpha$ and $B \in \beta$ in this list. Trace all the paths in the graph between the variables A and B. A path is blocked just in case it contains at least one serial node or divergent node which is in γ or at least one convergent node which is such that neither it nor any of its descendants are in γ. (A serial node is a node that has incoming and outgoing arrows on the path; a divergent node is a node that has two outgoing arrows on the path; a convergent node is a node that has two incoming arrows on the path). Check whether all the paths between A and B are blocked. If so, then move on to the next entry in the list. If for each entry in the list all the paths are blocked, then the conditional independence $\alpha\_||\_\beta|\gamma$ holds. (Pearl 1988: 116–118).

Estlund, D. (1994). Opinion leaders, independence and Condorcet's Jury Theorem. *Theory and Decision, 36*, 131–162.

Estlund, D. (1997). Beyond fairness and deliberation: The epistemic dimension of democratic authority. *Deliberative Democracy*, In: James Bohman and Willian Rehg (Eds.), Cambridge, Mass.: MIT Press, pp. 173–204.

Estlund, D. (1998). The insularity of the reasonable: Why political liberalism must admit the truth. *Ethics, 108*, 252–275.

Goldman, A.I. (1979). What is justified belief? In: G. S. Pappas (Ed.), *Justification and Knowledge*, Dordrecht: Reidel pp. 1–23.

Grofman, B., Owen G., & Feld, S.L. (1983). Thirteen theorems in search of the truth. *Theory and Decision, 15*, 261–278.

List, C., & Goodin, R.E. (2001). Epistemic democracy: Generalizing the Condorcet Jury Theorem. *Journal of Political Philosophy, 9*, 277–306.

Kornhauser, L.A. (1992a). Modelling collegial courts. I. path-dependence. *International Review of Law and Economics, 12*, 169–185.

Kornhauser, L.A. (1992b). Modelling collegial courts. II. Legal doctrine. *Journal of Law, Economics and Organization, 8*, 441–470.

Kornhauser, L.A., & Sager, L.G.(1986). Unpacking the court. *Yale Law Journal, 96*, 92–117.

Kornhauser, L.A., & Sager, L.G. (1993). The one and the many: Adjudication in collegial courts. *California Law Review, 81*, 1–59.

McLean, I. & Urken, A.B. (translators and editors) (1995). *Classics of social choice*. Ann Arbor: University of Michigan Press.

Owen, G., Grofman, B., & Feld, S.L. (1989). Proving a distribution-free generalization of the Condorcet Jury Theorem. *Mathematical Social Sciences, 17*, 1–16.

Pearl, J. (1988). *Probabilistic reasoning in intelligent systems: Networks of plausible inference*. 2nd ed. San Francisco, CA: Morgan Kaufmann.

Pettit, P. (2001). Deliberative democracy and the discursive dilemma. *Philosophical Issues, 11*(supplement 1 of *Nous, 35*), 268–295.

Pettit, P., & Rabinowicz, W. (2001). The Jury Theorem and the discursive dilemma. *Philosophical Issues, 11*(supplement 1 of *Nous 35*), appendix to Pettit (2001), pp. 295–299.

Rousseau, J.-J. 1997 (1762). The social contract. In: Victor Gourevich and Quentin Skinner (Eds.), *The 'Social Contract' and other later political writings,* Cambridge: Cambridge University Press.