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Judy Benjamin is a Sleeping Beauty

LUC BOVENS

Consider van Fraassen's (1981) Judy Benjamin (JB) problem. Judy is dropped in an area that is divided vertically in Blue (B) and Red (R) and horizontally in Headquarters (Q) and Second Company (S). These divisions define four quadrants, as in Figure 1 (roman script headings). Judy initially believes that there is an equal chance of being in each quadrant. She is then told by a fully reliable source that if she is in R, then there is a chance of $q > 0.50$ that she is in Q. Now ask yourself: what should Judy's credence be that she is in B on the basis of this information? I am interested here in the limiting case of this problem in which $q = 1$. Let us call this limiting case the JB′ problem.

Consider now Elga's (2000) Sleeping Beauty (SB) problem. Beauty is put to sleep on Sunday. A fair coin has been flipped. If Heads came up, then she will be awakened once, viz. on Monday (*Mo*). If Tails came up, she will be awakened twice, viz. on Monday and on Tuesday (*Tu*). After a Monday awakening, amnesia will be induced and she will have no memory on

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	B *Ta*	R *He*
Q *Mo*		
S *Tu*		X

Figure 1. The Common Structure of the JB′ and the SB′.

Tuesday of the awakening on Monday. Suppose Beauty knows all this from beforehand. Then upon an awakening, what is her credence for *Tails* – i.e. *P*(*Tails*)? Writers on the SB are notoriously split between ½-ers for whom *P*(*Tails*) = ½ and ⅔-ers for whom *P*(*Tails*) = ⅔.

Let us introduce a slight change in the story, which should not affect the value of *P*(*Tails*). Suppose that Beauty is first told that a fair coin was flipped and that there will be some awakenings on *Mo* or *Tu* (with 'or' denoting an inclusive or) and that amnesia is induced after awakenings. If that is all that she is being told on Sunday before being put to sleep, then one could reasonably stipulate that, upon an awakening, she thinks that there is an equal chance of ¼ for each combination, i.e. for *Heads&Mo*, *Heads&Tu*, *Tails&Mo* and *Tails&Tu*. But then, a minute after awakening, Beauty is informed of the actual set up, which is the same as in the original SB. As she is put back to sleep, amnesia is induced not only of the awakening but also of the information about the actual set up. On the background of her earlier information, the new information could be expressed as follows: if the coin came up *Heads*, then there is a chance of $q = 1$ that *Mo*. So what should her credence be for *Tails* – i.e. *P*(*Tails*) – on the basis of this new information? We call this the SB′ problem.

Now turn to Figure 1. Note the following structural similarity between JB′ and SB′. We start with a prior distribution in which each quadrant is equally probable. We then learn that, if the right value of the column variable holds, then the probability of the top value of the row variable is 1. The question in both JB′ and SB′ is, what should our posterior credence for the left value of the column variable be?

In the SB, ½-ers say that Beauty did not learn anything between Sunday and the awakening and so she has no reason to update her credence for *Tails*. And since she was told on Sunday that the coin is a fair coin, her credence upon awakening should still be ½. But note that, in the SB′, Beauty did learn something. So what kind of move could a ½-er make in the SB′? What one could say is something like this. Upon awakening, Beauty learns that

(*) If the coin came up *Heads*, then it must be *Mo*.

Now, granted, she did learn something, but somehow a ½-er needs to discount that she learned anything that could affect her credence for *Heads*. What a ½-er could say is that in learning a conditional, one does not learn anything about the truth of the antecedent and hence one's credence for the antecedent should remain unchanged. Bradley (2005: 351) claims that such a principle holds in a range of cases and dubs it 'Adams conditioning'. So $P(Heads) = ½$.

But this appeal to Bradley's Adams conditioning is problematic as the following reductio ad absurdum shows. Note that Beauty also learns upon awakening that

(#) If it is *Tu*, then the coin must have come up *Tails*.

She does not infer (#) from (*) by contraposition. Rather, when Beauty is told about the set up, then she learns (*) as well as (#). But then she should be willing to carry through the same reasoning on (#) as she did on (*). In learning the conditional sentence (#), she did not learn anything about the truth of the antecedent and hence her credence for the antecedent should remain unchanged. So $P(Tu) = ½$. But P(Heads) = ½ and P(Tu) = ½ cannot possibly be correct, as the following *reductio* shows. By the probability calculus, $P(Mo) = P(Mo|Heads)\ P(Heads) + P(Mo|Tails)\ P(Tails)$. Since $P(Tails) = 1–P(Heads) = ½$, $P(Mo) = 1–P(Tu) = ½$ and $P(Mo|Heads) = 1$, it follows that $P(Mo|Tails) = 0$. So when Beauty is being told *in addition* that *Tails* came up, then she could simply infer that it must be *Tu*. Obviously this inference is absurd. Hence, the appeal to Bradley's Adams conditioning in support of P(Heads) = ½ in the SB′ is unwarranted.

So what should we say about the SB′? Let us turn to the JB′ for help. The matrix in Figure 1 is fully symmetrical before the new information comes in – each column and each row have the same prior probability. Now the impact of excluding the south-eastern quadrant should have the same impact on the posterior credence for the row that does not include this quadrant as on the posterior credence for the column that does not include this quadrant. There is nothing special about rows *versus* columns before the new information comes in and the only thing that sets apart the bottom row and the left column is the shared feature that they do not cover the excluded quadrant. Hence, by symmetry,

(1) $P(B) = P(Q)$

Since the information is fully reliable,

(2) $P(Q|R) = 1$

and since no information was provided about Judy's location if she is in B, her prior credence for being in Q if she is in B remains unchanged,

(3) $P(Q|B) = ½$

By the probability calculus,

(4) $P(Q) = P(Q|B)P(B) + P(Q|R)(1\text{-}P(B))$

Hence, from (1) to (4),

(5) $P(B) = 2/3$

(5) is also van Fraassen's (1981: 379) solution to JB′, i.e. in the limiting case of JB in which q goes to 1.

Similarly, we can argue that $P(Tails) = 2/3$ in SB′. The argument starts from the assumption that since the new information in SB′ only affects the non-existence of the *Heads&Tu* combination, the information should have the same impact on the posterior credence for *Tails* as it has on the posterior credence for *Mo* – *cf.* (1). Clearly, SB′ can infer that *Mo* when she is told that it is *He* – *cf.* (2). And if she is told that it is *Ta*, then she would still assign probability ½ to *Mo* – *cf.* (3). And these assumptions are sufficient to infer that $P(Tails) = 2/3$ – *cf.* (5). Since it should not matter whether the set up is revealed on Sunday (as in the SB) or upon awakening (as in the SB′), P(Tails) = ⅔ is also is also the solution to the SB.

In conclusion, a close variant of Elga's Sleeping Beauty problem (SB′) and a limiting case of van Fraassen's Judy Benjamin problem (JB′) have the same structure. I presented an argument for ½-ers in the SB′. But this argument turns out to be fallacious. A symmetry argument in JB′ can be invoked to show that Judy Benjamin's posterior credence that she is in B is ⅔. And by parity of reasoning, it can be shown that $P(Tails) = 2/3$ is the correct solution to the SB′ and hence to the SB.[1]

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