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SEQUENTIAL COUNTERFACTUALS, COTENABILITY AND TEMPORAL BECOMING*

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I. SEQUENTIAL COUNTERFACTUALS AND TIME-RELATIVE TRUTH

It has received little attention that the truth values of sequential counterfactuals (i.e. counterfactuals in which the antecedent event precedes the consequent event) can shift over time. In the footsteps of Goodman, I develop a theory of sequential counterfactuals that can account for this feature. The theory (i) defends a semifactual test of cotenability, (ii) spells out the truth conditions for semifactuals in probabilistic terms and (iii) accounts for truth-value shifts by appealing to a dynamic view of time.

Suppose that Johann is deliberating whether to invite Bettina (at time t) to come to a party (at a later time t′) and that Bettina will come if and only if she is invited by Johann. He decides not to invite her and consequently Bettina does not come to the party. The challenge for truth conditional theories of counterfactuals consists in spelling out the truth conditions for a proposition that takes on, say, the following formal expression:

(*CF*) Invite(Johann, Bettina, t) $\Box\!\!\rightarrow$ Come(Bettina, t′)

But, depending on the time of utterance, this proposition is expressed by different sentences in English. Suppose that Johann is deliberating whether he will invite Bettina to the party and decides not to invite her. At this point in time, he says to himself:

(*CF*′) If I were to invite Bettina to the party, then she would come to the party.

After having foregone the opportunity to invite Bettina but before the party, Johann says to himself:

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(CF'') If I had invited Bettina to the party, then she would come to the party.

The day after the party, he says to himself:

(CF''') If I had invited Bettina to the party, then she would have come to the party.

It is a curious feature about sequential counterfactuals that their truth values may shift as time progresses. To see this, let us first consider a case involving a semifactual – i.e. a counterfactual with a true consequent. Both Johann and Ludwig are inviting friends to a joint party. Johann is trying to decide whether to invite Bettina (at time t) or not. He knows that Ludwig will flip a coin tomorrow and invite Bettina if and only if heads comes up. Bettina will come to the party (at a later time t′) if and only if she gets an invitation from either Johann or Ludwig. Now, Johann does invite Bettina. Suppose that he says at the time of inviting her:

(SFi) Even if I were not to invite Bettina, she would still come to the party.

Or, suppose that after inviting Bettina, but before Ludwig's coin toss, he says:

(SFii) Even if I had not invited Bettina, she would still come to the party.

Ludwig's coin comes up heads and he invites Bettina. Suppose that, after Ludwig's coin toss but before the party, Johann says:

(SFiii) Even if I had not invited Bettina, she would still come to the party.

Bettina comes to the party. Suppose that, after the party, Johann says:

(SFiv) Even if I had not invited Bettina, she would still have come to the party.

Clearly, (SFiii) and (SFiv) are true. Even if Johann had not invited Bettina, she would still come or have come to the party, since Ludwig invited her. But what about (SFi) and (SFii)? Before Ludwig's coin toss, the coin might or might not have come up heads and so Ludwig might or might not invite Bettina to the party. Consequently, were Johann not to invite Bettina or had Johann not invited Bettina, then she might or might not come to the party. But, following Lewis[1], if she might not come to the party, then it is not true that she would come to the party and so, (SFi) and (SFii) are false. Hence, the truth value of the semifactual:

(SF) not-Invite(Johann, Bettina, t) $\Box\!\!\rightarrow$ Come(Bettina, t$'$)

switches at the time of Ludwig's coin toss.

Let us now turn to a truth value switch over time for a counterfactual that is not a semifactual. Johann and Ludwig are inviting friends for a joint party and they are both keen on winning Bettina's favors at the party. Bettina will come to the party if and only if she gets an invitation from either Johann or Ludwig. Bettina is amorous but hates to be pursued: if she comes to the party, she will bestow her favors on a party host if and only if he did not invite her to the party. Both Johann and Ludwig prefer Bettina's coming to the party and bestowing her favors on them to Bettina's coming to the party and not bestowing her favors on them to Bettina's not coming to the party. Johann is trying to decide whether to invite Bettina or not. He knows that Ludwig will flip a coin tomorrow and invite her if and only if heads comes up. Johann does invite Bettina. Suppose that, at the time of inviting her, he says:

(CFi) If I were not to invite Bettina, then she would bestow her favors on me at the party.

Or suppose that, after having invited Bettina but before Ludwig's coin toss, Johann says:

(CFii) If I had not invited Bettina, then she would bestow her favors on me at the party.

Ludwig's coin comes up heads and he invites Bettina to the party. Suppose that, after Ludwig's coin toss but before the party, Johann says:

(CFiii) If I had not invited Bettina, then she would bestow her favors on me at the party.

Bettina comes to the party but does not bestow her favors on either one of them. Suppose that, after the party, Johann says:

(CFiv) If I had not invited Bettina, then she would have bestowed her favors on me at the party.

Clearly, (CFiii) and (CFiv) are true. If Johann had not invited Bettina, Bettina would have come on Ludwig's invite and she would have bestowed her favors on Johann at the party. But what about (CFi) and (CFii)? Before Ludwig's coin toss, the coin might or might not come up heads and so Ludwig might or might not invite Bettina. Consequently, were Johann not to invite Bettina or had Johann not invited Bettina, then she might or might not come to the party and hence she might or might not bestow her favors on Johann at the party. But if she might not bestow her favors on Johann at the party, then it is not true that she would bestow her favors on Johann at the party. Hence, (CFi) and (CFii) are false. The truth value of the counterfactual:

(CF) not-Invite(Johann, Bettina, t) $\Box\!\!\rightarrow$ Bestow(Bettina, Johann, t')

switches at the time of Ludwig's coin toss.[2]

In writing about counterfactuals, a common problem of presentation surfaces. Either one checks intuitions by means of examples involving electrical circuits (or some other well-understood physical system) or by means of examples involving human interactions. The former has the advantage that the issues can be presented in a very crisp manner: one presents a set up involving certain inputs, outputs and chance mechanisms and certain well-known laws of nature come into play in the operation of the system. The disadvantage is that it makes for highly artificial problems such that, even if the reader is able to overcome his sense of boredom, intuitions start giving out fast. The latter has the advantage that it is possible to keep the reader sufficiently engaged to have firm intuitions about the matter. It has the disadvantage however that it deals with the less understood issue of human agency.

Consider the following circuit which is structurally analogous to our last example. Imagine a set up in which Green can push a button at 9 o'clock and Red can push a button at 8 o'clock. Green's button emits 25 volts to a light bulb. Red's button engages a chance mechanism at 10 o'clock that either emits 40 volts to the same light bulb with a probability of 0.50 or blocks the current with a probability of 0.50. The light bulb will glow red at 25 volts, green at 40 volts and white at 65 volts at 11 o'clock. Suppose both Green and Red push their respective buttons, the chance mechanism transmits the current and the light bulb glows white.[3] Just like in our original example, Green can utter the following series of counterfactuals:

> at 9:00: (CFi′) If I were not to push my button, then the light bulb would glow green;
>
> at 9:01: (CFii′) If I had not pushed my button, then the light bulb would glow green;
>
> at 10:01: (CFiii′) If I had not pushed my button, then the light bulb would glow green;
>
> at 11:01: (CFiv′) If I had not pushed my button, then the light bulb would have glowed green.

Just like in our original example, (CFi′) and (CFii′) are false and (CFiii′) and (CFiv′) are true.

I want to secure the best of both worlds by presenting my cases by means of examples that involve human interactions while assigning a motivational structure to the players that must be understood to have the same precision as the set up of an electrical circuit. E.g., if a player is disposed to react in some way when prompted in some manner, then the laws of nature ensure that he will react in this way when prompted in this manner, just as, if a light bulb is constructed to glow a certain color when triggered in some way, then the laws of nature ensure that the light will glow a certain color when triggered in this way. This idealization will do no injustice to the problem at hand and is indispensable in constructing a precise analysis of counterfactual statements.

II. GOODMAN'S COTENABILITY PROBLEM

Goodman's attempt to spell out the truth conditions for counterfactuals notoriously runs into an impasse. On his account, a counterfactual is true if and only if its antecedent in conjunction with the laws of nature and *some* true statements of fact entail its consequent. But which true statements of fact should be retained?

Suppose a particular match was not struck. It was (i) well-constructed, (ii) dry, (iii) surrounded by oxygen and (iv) did not light. Now consider the counterfactual:

(CF1) If the match had been struck, it would have lit.

Intuitively, this counterfactual is true. And indeed, there exists a law of nature, viz.

(L) Matches that are dry, well-constructed and surrounded by oxygen will light if scratched

such that the antecedent of (CF1) in conjunction with the true statements of fact (i), (ii) and (iii) and (L) entails the consequent of (CF1). But now consider the following counterfactuals:

(CF2) If the match had been struck, it would not have been dry.

(CF3) If the match had been struck, it would not have been well-constructed;

(CF4) If the match had been struck, it would not have been surrounded by oxygen.

Intuitively, these counterfactuals are false. But notice that the antecedents of each of these counterfactuals in conjunction with (iv), some combination of (i), (ii) and (iii), and (L) entail their respective consequents.

The analysis yields correct truth-values as long as we include no more than the true statements of fact (i), (ii) and (iii) in the inference base. But it yields an incorrect truth-value whenever we include the true statement of fact (iv) as well in the inference base. Hence, the culprit seems to be the inclusion of (iv) – i.e. the true statement of fact that the match did not light – in the inference base. But what is the difference between the true statements of fact (i), (ii) and (iii) on the one hand and the true statement of fact (iv) on the other hand? Can the analysis be refined by determining some principled way of including true statements of fact such as (i), (ii) and (iii) in the inference base while excluding true statements of facts such as (iv) from the inference base?

Goodman distinguishes between true statements of facts that are *cotenable* with the antecedent and true statements of fact that are not cotenable with the antecedent. It is cotenable with the match's being struck that it is dry, well-constructed and surrounded by oxygen. But it is not cotenable with the match's being struck that it did not light. The inference base should only include true statements of facts that are cotenable with the antecedent. But what is it for a true statement of fact to be cotenable with the antecedent? Goodman proposes the following criterion. A true statement of fact T is cotenable with the antecedent A of a counterfactual if and only if it is false that if A had occurred, T would not have occurred. I will name this the *negative counterfactual test of cotenability*. Let us apply the test to the example at hand. Consider the following counterfactuals:

(CFi) If the match had been struck, it would not have been well-constructed;

(CFii) If the match had been struck, it would not have been dry;

(CFiii) If the match had been struck, it would not have been surrounded by oxygen;

(CFiv) If the match had been struck, it would not have been the case that it did not light (i.e. it would have lit).

Since (CFi), (CFii) and (CFiii) are false, (i), (ii) and (iii) are true statements of fact that are cotenable with the antecedent and since (CFiv) is true, (iv) is a true statement of fact that is not cotenable with the antecedent.

We can thus present the following analysis:

(A)(i) a counterfactual is true if and only if its antecedent in conjunction with the laws of nature and true statements of fact that are cotenable with the antecedent entail its consequent.

(ii) a true statement of fact T is cotenable with the antecedent if and only if T meets the negative counterfactual test.

This analysis will provide the correct truth values for (CF1) to (CF4), since it provides a principled way to include (i), (ii) and (iii) in the inference base and to exclude (iv) from the inference base.

But, as Goodman acknowledges, this analysis leads to an impasse. The analysis of counterfactuals includes a reference to counterfactuals. This leads us, in Goodman's words, into 'a circle; for cotenability is defined in terms of counterfactuals, yet the meaning of counterfactuals is determined in terms of cotenability'. (1991: 19)

I will try to design a theory that breaks out of Goodman's circle. But in order to do so, we will first need to take a critical look at the negative counterfactual test of cotenability. Goodman stresses that it is the negative counterfactual test that provides for an accurate assessment of cotenability and is followed by Bennett (1974: 390) in this lead. Kvart, on the other hand, proposes a positive semifactual test of cotenability:

(ii′) A true statement of fact T is cotenable with the antecedent A of a counterfactual if and only if, even if A had occurred, T would have occurred. (1986: 63)

To add to the confusion, Goodman's position on cotenability is often misreported. Both Tichy (1984: 150) and Horwich (1987: 158) take Goodman to hold the positive semifactual test of cotenability. What is at stake here?

III. NEGATIVE COUNTERFACTUALS OR POSITIVE SEMIFACTUALS?

a. *Conditional Excluded Middle.* Stalnaker (1980) and Lewis (1973: 77-83) disagree notoriously about the validity of the law of conditional excluded middle, i.e.

(i) $(P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \neg Q)$

On Stalnaker's theory, but not on Lewis' theory, this law is valid. For Stalnaker, a counterfactual $P \Box\!\!\rightarrow Q$ is true if and only if Q is true at *the* closest possible P-world. Now either Q is true or Q is false at this world. If Q is true, then $P \Box\!\!\rightarrow Q$ and, if Q is false, then $P \Box\!\!\rightarrow \neg Q$. Hence, the law of conditional excluded middle holds. For Lewis, there can be multiple closest possible worlds. The counterfactual $P \Box\!\!\rightarrow Q$ is true if and only if some P&Q-world is a closer possible world than any P&¬Q-world. Similarly, the counterfactual $P \Box\!\!\rightarrow \neg Q$ is true if and only if some P&¬Q-world is a closer possible world than any P&Q-world. Now suppose that the closest P&Q-worlds and the closest P&¬Q-worlds are equally close. Then there is neither a P&Q-world which is a closer possible world than any P&¬Q-world, nor is there a P&¬Q-world which is a closer possible world than any P&Q-world. So, neither $P \Box\!\!\rightarrow Q$ is true, nor $P \Box\!\!\rightarrow \neg Q$ is true. Hence, the law of conditional excluded middle does not hold up.

Furthermore, it is agreed that $P \Box\!\!\rightarrow Q$ and $P \Box\!\!\rightarrow \neg Q$ cannot both be true:

(ii) $\neg[(P \Box\!\!\rightarrow Q)\&(P \Box\!\!\rightarrow \neg Q)]$

For Stalnaker, Q and ¬Q cannot both be true in the closest possible P-world and so (ii) is a valid law. For Lewis, it cannot be the case that both some P&Q-world is a closer possible world than any P&¬Q-world *and* that some P&¬Q-world is a closer possible world than any P&Q-world and so (ii) is a valid law. The conjunction of (i) and (ii) is equivalent to

(iii) $(P \Box\!\!\rightarrow Q) \leftrightarrow \neg(P \Box\!\!\rightarrow \neg Q)$

and so the distinction between the positive semifactual test and the negative counterfactual test becomes a non-issue. However, if, with Lewis, we do not accept (i), then one direction of the biconditional fails and the distinction remains a genuine distinction.

I will not entangle myself in the debate between Lewis and Stalnaker about the law of conditional excluded middle. Rather, let us consider the following question. If the law of conditional excluded middle fails and hence the distinction between the positive semifactual test and the negative counterfactual test is a genuine distinction, then should cotenability be determined by means of the former or the latter?

b. *In Defense of the Positive Semifactual Test.* Suppose that Bettina will come to the party if and only if she is invited by *both* Johann and Ludwig. If Johann does not invite Bettina, then Ludwig is determined to invite Bettina. But if Johann does invite Bettina, then Ludwig's determination dwindles: instead, he will toss a coin and invite her if and only if heads comes up. Suppose Johann does not invite Bettina. Hence Ludwig and only Ludwig invites Bettina without a coin toss and Bettina will not come to the party. Now consider the following counterfactual:

(CF1) If Johann had invited Bettina, then she would have come to the party.

This counterfactual is false. If Johann had invited Bettina, Ludwig would have tossed a coin and he might or might not have invited Bettina and so Bettina might or might not have come to the party. Hence, it is neither true that Bettina would not have come to the party, had Johann invited Bettina, nor is it true that Bettina would have come to the party, had Johann invited Bettina.

Now consider the true statement of fact that Ludwig invited Bettina. The antecedent of the counterfactual in conjunction with this true statement of fact, a description of the dispositions of each player and the laws of nature entails Bettina's appearance at the party. Hence, to keep (CF1) from being true, Ludwig's invitation must *not* be cotenable with the antecedent of the counterfactual.

We compare the positive semifactual test and the negative counterfactual test:

(SF^{pos}) Even if Johann had invited Bettina, Ludwig would still have invited Bettina.

(CF^{neg}) It is not the case that if Johann had invited Bettina, then Ludwig would not have invited Bettina.

Clearly, if Johann had invited Bettina, then Ludwig might or might not have invited Bettina. Hence, it is not the case that, if Johann had invited Bettina, Ludwig would not have invited Bettina – i.e. (CF^{neg}) is true – and it is not the case that, if Johann had invited Bettina, Ludwig would have invited Bettina – i.e. (SF^{pos}) is false. Since (CF^{neg}) is true, Ludwig's invitation is cotenable with the antecedent of the counterfactual on the negative counterfactual test and since (SF^{pos}) is false, Ludwig's invitation is not cotenable with the antecedent of the counterfactual on the positive semifactual test. Since non-cotenability is the desired result, it is thus the positive semifactual test and not the negative counterfactual test that provides an adequate test of cotenability.

c. *If-Semifactuals vs. Even-If Semifactuals.* What motivated Goodman (1991) and Bennett (1974) to embrace the negative counterfactual test of cotenability? Goodman and Bennett distinguish between the truth conditions of *even-if* semifactuals and *if*-semifactuals. They acknowledge that the positive *if*-semifactual test is too strict and, to compensate for this, they resort to the weaker negative counterfactual test without realizing that this test is too permissive.

Suppose that Bettina went to a party knowing that neither Johann nor Ludwig would be there. Suppose furthermore that Bettina has a great liking for Johann but not for Ludwig. Bettina might utter the following semifactuals:

(i) Even if Ludwig had been at the party, I would (still) have gone;

(ii) If Johann had been at the party, I would (most certainly) have gone.

Semifactual (ii), unlike semifactual (i), signals that there is some positive influence from the antecedent to the consequent. Now, we may want to build the presence of such an influence into the truth conditions of *if*-semifactuals. If we follow this line, then the *if*-semifactual is indeed a poor candidate for the positive semifactual test of cotenability. Suppose that both Ludwig and Johann have the opportunity to invite Bettina *independently* to a joint party and that Bettina will come to the party if and only if she gets an invitation from both Ludwig and Johann. Actually, Ludwig does not invite Bettina, Johann does invite Bettina and Bettina does not come to the party. Clearly, the following counterfactual is true:

(CF2) If Ludwig had invited Bettina, then Bettina would have come to the party.

Now, consider the semifactuals:

(i) Even if Ludwig had invited Bettina, Johann would (still) have invited Bettina.

(ii) If Ludwig had invited Bettina, Johann would (most certainly) have invited Bettina.

One might contend that (i) is true while (ii) is not, since Ludwig's inviting Bettina does not exert any positive influence on Johann's inviting Bettina. Hence, Johann's invitation is not cotenable on the positive *if*-semifactual test and is cotenable on the positive *even-if* semifactual test. But Johann's invitation must come out to be cotenable with the antecedent of (CF2): (CF2) is a true counterfactual because the antecedent of (CF2) in conjunction with the laws of nature, the true statement of fact that Johann invited Bettina and true statements of fact about the dispositions of each player entails Bettina's presence at the party. Hence the positive *if*-semifactual test fails.

Lewis (1973: 33n) and Hazen and Slote (1979) disagree with Goodman (1991) and with Bennett (1974) that the truth-conditions for *if*-counterfactuals and *even-if* counterfactuals are different and argue that there is no truth-conditional difference but merely a difference in conversational implicatures. In his later work, Bennett (1982) changes his mind and denies as well that there are any truth-conditional differences between *if*-semifactuals and *even-if* semifactuals.

The question whether the difference between *if*-semifactuals and *even-if* semifactuals is truth-conditional or not does not concern me here. If it is, then a true statement of fact is cotenable if and only if it passes the positive *even-if* semifactual test.[4] If it is not, then a true statement of fact is cotenable if and only if it passes the positive semifactual test *tout court*. Whatever route we choose to take, the upshot is that it is a mistake to think that we can substitute the negative counterfactual test of cotenability. The positive *even-if* semifactual test (or the positive semifactual test *tout court*) is stricter than the negative counterfactual test and it is the stricter test which is needed in the analysis of counterfactuals. I can now propose the following analysis:

(A′)(i) A counterfactual is true if and only if its antecedent in conjunction with the laws of nature and true statements of fact that are cotenable with the antecedent entail its consequent.

(ii′) A true statement of fact T is cotenable with the antecedent A of a counterfactual if and only if, (even) if A had occurred, T would have occurred.

IV. A PROBABILISTIC THEORY OF SEMIFACTUALS

An analysis of counterfactuals in terms of positive semifactuals is of course no less circular than an analysis in terms of negative counterfactuals, since semifactuals are just a special type of counterfactuals. Can we break out of this circle? A positive semifactual holds true if and only if the switch from the antecedent event's not obtaining to the antecedent event's obtaining stands in a particular relation to the consequent event's obtaining. I will present a probabilistic analysis of this relation.[5] Let W be the set of events prior to or at the time of the antecedent event, but excluding the denial of the antecedent event.[6] What comes to mind is the following proposal: the semifactual A□→C is true if and only if

(*) $P(C|A\&W) \geq P(C|\neg A\&W)$

But (*) provides neither a sufficient nor a necessary condition. To see that (*) does not provide a sufficient condition, consider the following case. Johann is deliberating whether to invite Bettina to a party. He initially resolves to cast a die and to invite her if and only if the die comes up six, but then he changes his mind and decides to spin a roulette wheel and to invite her if and only if the ball lands on a six. Bettina will come to the party if and only if she is invited by Johann. Johann spins the roulette wheel, the ball lands on six, Johann invites Bettina to the party and Bettina comes to the party. Now suppose that Johann says:

(SF1) Even if I had stuck to my initial resolution (A), Bettina would still have come to the party (C).

Clearly this semifactual is false. There is no telling whether the die would have come up six, had Johann cast a die rather than spun a roulette wheel. But notice that

$$P(C|A\&W) = 1/6 \geq 1/38 = P(C|\neg A\&W)$$

Hence, (*) is not a sufficient condition.

To see that (*) does not provide for a necessary condition, consider the following case. Johann and Ludwig can invite Bettina to a party and Bettina will come to the party if and only if she is invited by either one of them. Johann invites Bettina on Monday. Independently of Johann's action, Ludwig resolves on Sunday to toss a coin on Tuesday. He will invite Bettina if and only if the coin comes up heads. Ludwig's coin comes up heads on Tuesday and he invites Bettina. Bettina comes to the party. Suppose that Johann says:

(SF2) Even if I had not invited Bettina (A), she would still have come to the party (C).

Clearly, this semifactual is true. Even if Johann had not invited Bettina, she would still have come to the party on Ludwig's invite. But notice that

$$P(C|A\&W)= 1/2 < 1 = P(C|\neg A\&W)$$

Hence, (*) is not a necessary condition.

A more sophisticated account is needed. The switch from an actual event to a counterfactual event sets the world on a different track. Certain actual events will remain and others will vanish on this track and certain non-actual events are bound to emerge on this track. The question is whether some later actual event would indeed be preserved on this different track. A probabilistic model can be developed to answer this question.

The intuitive idea is as follows. Divide the time span between the antecedent time and the consequent time into separate time intervals. For the antecedent time, the consequent time and each time that demarcates between two time intervals, construct the set of actual events at or prior to this time. Let us name this *the set of actual events connected to this time*. Consider the set of actual events at or prior to the antecedent time. Substitute the antecedent's obtaining for the antecedent's not obtaining within this set. Let us call this newly constructed set *the set of counterfactual events connected to this time*. Now consider the time that demarcates the first time interval from the second time interval. Take each actual event in this first time interval and ask yourself the following question. Is the conditional probability of this actual event on the set of counterfactual events connected to the preceding time (*in casu* the antecedent time) at least as great as the conditional probability of this actual event on the set of actual events connected to the preceding time (*in casu* the antecedent time)? If so, retain the event and if not, delete the event. Furthermore, add all the possible events that are entailed by the counterfactual set of events connected to the preceding time through the laws of nature. The newly constructed set of events is the counterfactual set of events connected to this time. The procedure is repeated until we reach the latest time interval – i.e. the time interval containing the consequent of the semifactual. The procedure will yield different results according to how the time span between the antecedent and the consequent was divided. A semifactual is true if and only if the consequent event is contained in the set of counterfactual events connected to the consequent time on the finer divisions of the time span between the antecedent event and the consequent event.

Let us work out a formal model to capture this intuitive idea. Let $\neg$A and C be actual events such that $t_{\neg A} \prec t_C$ – i.e. $t_{\neg A}$ is earlier than t_C. Let L be the laws of nature. Consider a semifactual A$\Box\!\!\rightarrow$C. Let T be some time series $\langle$t0, t1, t2, ..., tn$\rangle$ for $t_{\neg A}$=t0, t_C=tn and ti$\prec$ti+1. T$'$ is a *refinement* of T if and only if (i) all ti contained in T are also contained in T$'$ and (ii) there are at least two consecutive elements ti and tk in T for which there exists an element tj in T$'$ such that ti$\prec$tj$\prec$tk. Take an arbitrary time series T. Let $\mathcal{E}_{t0}{}^{A}$ be the set of events prior to t0 or at t0, substituting A for $\neg$A. Construct $\mathcal{E}_{tn}{}^{A}$ recursively as follows: for each ti (with i=0, ..., n$-$1) in T, construct $\mathcal{E}_{t(i+1)}{}^{A}$ by enriching $\mathcal{E}_{ti}{}^{A}$ with (i) the actual events during the half-open time interval (ti, t(i+1)] such that their probability of occurring given $\mathcal{E}_{ti}{}^{A}$ is greater than or equal to their probability of occurring given $\mathcal{E}_{ti}{}^{@}$ (i.e. the set of actual events prior to ti or at ti) and with (ii) the possible events during (ti, t(i+1)] that are entailed by ($\mathcal{E}_{ti}{}^{A}$&L). Let us denote $\mathcal{E}_{tn}{}^{A}$ constructed on grounds of the time series T by '$\mathcal{E}_{tn}{}^{A}(T)$'. The semifactual A$\Box\!\!\rightarrow$C is true if and only if there exists a T such that $\mathcal{E}_{tn}{}^{A}(T)$ contains C and there does not exist a refinement T′ of T such that $\mathcal{E}_{tn}{}^{A}(T')$ does not contain C.

As an illustration, let us consider how the analysis deals with the case in which Johann switches from a die to a roulette wheel in order to decide whether to invite Bettina. We will consider the following actual and possible events: ¬A: Johann's changing his mind; e1: Johann's spinning a roulette wheel; e2: Johann's casting a die; e3: the ball's rolling into the six; e4: the die coming up six; e5: Johann's inviting Bettina; C: Bettina's coming to the party. I will show that (SF1) is a false semifactual on my analysis.

Let us assume that, had Johann decided to cast a die, then he would have cast the die at the same time as when he actually spun the roulette wheel, and let us assume that the rolling of the die would occupy the same time span as the actual spinning of the roulette wheel. Hence, $t_{e1}=t_{e2}$ and $t_{e3}=t_{e4}$. Neither one of these assumptions are needed to obtain the desired result, but they simplify the analysis.

Let T be the highly refined time series $\langle t_{\neg A}, t_{e1}, t_{e3}, t_{e5}, C\rangle$. Let us incrementally construct $\mathcal{E}_{t_C}{}^{A}$:

(i) the event A is included in $\mathcal{E}_{t_{\neg A}}{}^{A}$;
(ii) the actual event e1 is not included in $\mathcal{E}_{t_{e1}}{}^{A}$, since, considering e1's status as an actual event, $P(e1|\mathcal{E}_{t_{\neg A}}{}^{A}) = 0 < 1 = P(e1|\mathcal{E}_{t_{\neg A}}{}^{@})$: the probability of Johann's spinning the roulette wheel, given that he decided to cast a die, is lower than the probability of Johann's spinning the roulette wheel, given that he decided to spin the roulette wheel; and, since, considering e1's status as a possible event, it is not entailed by $(\mathcal{E}_{t\neg A}{}^{A}\&L)$.[7]
(iii) the possible event e2 is included in $\mathcal{E}_{t_{e1}}{}^{A}$, since e2 is entailed by $(\mathcal{E}_{t_{\neg A}}{}^{A}\&L)$: Johann's deciding to cast a die in conjunction with the laws of nature entails his casting a die.
(iv) the actual event e3 is not included in $\mathcal{E}_{t_{e3}}{}^{A}$, since, considering e3's status as an actual event, $P(e3|\mathcal{E}_{t_{e1}}{}^{A}) = 0 < 1/38 = P(e3|\mathcal{E}_{t_{e1}}{}^{@})$: the probability of the ball rolling into the six on the roulette wheel, given that Johann cast a die, is lower than the probability of the ball rolling into the six, given that Johann spun the roulette wheel; and since, considering e3's status as a possible event, it is not entailed by $(\mathcal{E}_{t_{e1}}{}^{A}\&L)$.
(v) the possible event e4 is not included in $\mathcal{E}_{t_{e3}}{}^{A}$, since e4 is not

entailed by ($\mathcal{E}_{t_{e1}}{}^{A}$&L): Johann's die coming up six is not entailed by Johann's casting a die in conjunction with the laws of nature.
(vi) the actual event e5 is not included in $\mathcal{E}_{t_{e5}}{}^{A}$, since, considering e5's status as an actual event, $P(e5|\mathcal{E}_{t_{e3}}{}^{A}) = 1/6 < 1 = P(e5|\mathcal{E}_{t_{e3}}{}^{@})$: the probability of Johann inviting Bettina, given that he decided to cast a die and to invite Bettina if and only if the die would come up six and that he cast a die, is lower than the probability of Johann inviting Bettina, given that he decided to spin a roulette wheel and to invite Bettina if and only if the ball would roll into the six and that he spun the roulette wheel *and the ball did roll into the six*; and since, considering e5's status as a possible event, it is not entailed by ($\mathcal{E}_{t_{e3}}{}^{A}$&L).
(vii) the actual event C is not included in $\mathcal{E}_{t_{C}}{}^{A}$, since, considering C's status as an actual event, $P(C|\mathcal{E}_{t_{e5}}{}^{A}) = 1/6 < 1 = P(C|\mathcal{E}_{t_{e5}}{}^{@})$: the probability of Bettina coming to the party, given that Johann's inviting Bettina is contingent on a cast of the die and that she will come to the party if and only she is invited by Johann, is lower than the probability of Bettina coming to the party, given that she was invited by Johann and that she will come to the party if and only she is invited by Johann; and since, considering C's status as a possible event, it is not entailed by ($\mathcal{E}_{t_{e5}}{}^{A}$&L).
Since the actual event C is not included in $\mathcal{E}_{t_{C}}{}^{A}$, (SF1) is false.

Note the importance of choosing a refined time series. Suppose we would carry out the analysis for the time series $\langle t_{\neg A}, t_C\rangle$. On this time series, C is included in $\mathcal{E}_{t_{C}}{}^{A}$, since $P(C|\mathcal{E}_{t_{\neg A}}{}^{A}) = 1/6 \geq 1/38 = P(C|\mathcal{E}_{t_{\neg A}}{}^{@})$: the probability of Bettina's coming to the party, given that Johann decides to cast a die and to invite Bettina if and only if the die comes up six, is greater than or equal to the probability of Bettina's coming to the party, given that Johann decides to spin the roulette wheel and to invite Bettina if and only if the ball rolls into the six. But this does not make the semifactual come out true, since there exists a more refined series, viz. $\langle t_{\neg A}, t_{e1}, t_{e3}, t_{e5}, C\rangle$, on which C is not included in $\mathcal{E}_{t_{C}}{}^{A}$.

V. TEMPORAL BECOMING

Let us now return to (CFi) to (CFiv) in section I. To begin with, I will determine the truth values of these counterfactuals on grounds of analysis (A′) – i.e. the cotenability account of counterfactuals combined with the positive semifactual account of cotenability – in section III. Subsequently, I will determine the truth values of the relevant semifactuals – i.e. (SFi) to (SFiv) in section I – on grounds of my probabilistic theory of semifactuals and a dynamic view of time and I will evaluate my theory in the light of two competing dynamic views of time.

a. *From Semifactuals to Counterfactuals.* I contended earlier that (CFi) and (CFii) are false while (CFiii) and (CFiv) are true. Analysis (A′) respects these truth values. Consider (CFi) and (CFii). Before Ludwig's coin flip, the semifactuals (SFi) and (SFii) come out false: if Johann would not or had not invited Bettina, then Bettina might not be present at the party and hence it is not true that she would be present at the party. Hence, on the positive semifactual test, Bettina's presence at the party is not cotenable with the antecedent of (CFi) and (CFii). The antecedents of (CFi) and (CFii), in conjunction with the laws of nature and cotenable statements of fact about the dispositions of each player, do not entail their consequents, since Johann's not inviting Bettina is consistent with Bettina's absence at the party and consequently with Bettina's not bestowing her favors on Johann at the party. Hence, (CFi) and (CFii) are false. Consider (CFiii) and (CFiv). After Ludwig's coin flip, the semifactuals (SFiii) and (SFiv) come out true: even if Johann had not invited Bettina, she would still be present at the party since Ludwig's coin came up heads. Hence, on the positive semifactual test, Bettina's presence at the party is cotenable with the antecedents of (CFiii) and (CFiv). The antecedents of (CFiii) and (CFiv), in conjunction with the laws of nature, Bettina's presence at the party and cotenable true statements of fact about the dispositions of each player entail their consequents, since Bettina will bestow her favors on any non-inviting host at the party. Hence, (CFiii) and (CFiv) are true.

b. *Temporal Becoming and the Probabilistic Theory of Cotenability.* In determining the truth values of a semifactual $A \Box\!\!\rightarrow C$ we construct a set $\mathcal{E}_{t_C}{}^{A}$. This construction is contingent on the contents of $\mathcal{E}_t{}^{@}$ for all t contained in some sufficiently refined time series $\langle t_{\neg A}, \ldots, t_C \rangle$. If it is the case that the content of some particular $\mathcal{E}_t{}^{@}$ is in flux over time, then it is not surprising that $\mathcal{E}_{t_C}{}^{A}$ will be in flux over time and hence that the truth value of the semifactual A□→C will be in flux over time.

Consider a particular time t. Clearly, at a time t+, later than or contemporaneous with t, $\mathcal{E}_t{}^{@}$ contains all the actual events prior to t or at t. But at some time t−, earlier than t, a defender of a dynamic view of time will contend that $\mathcal{E}_t{}^{@}$ cannot contain all the actual events prior to t or at t. Consider the following quote by Peirce:

> A certain event either will happen or it will not. There is nothing now in existence to constitute the truth of its being about to happen, or of its being about not to happen, unless it be certain circumstances to which only a law or uniformity can lend efficacy. (. . .) If (. . .) we admit that the law has a real being, not of the mode of being of an individual, but even more real, then the future necessary consequent of a present state of thing is as real and true as that present state of things itself. (1965: 6.368 – reference from Rescher (1967: 214))

Peirce assigns existence to all past and present events and to all future events that are entailed by past and present events through the laws of nature. He withholds existence from future contingent events. Past and present events acquire their existence directly through having been touched by the gnawing tooth of time or, in other words, through having temporally become. Future necessary events acquire their existence indirectly through being entailed by existing events and existing laws of nature. But future contingent events have no grounds to base their existence upon.

Following Peirce, let $\mathcal{E}_t{}^{@}$ at t− include the past and present events at t− and all the future events in the half-open time interval (t−,t] that are entailed by the past and present events at t− through the laws of nature, but not the future contingent events in the half-open time interval (t−,t]. As the present moves up to time t, $\mathcal{E}_t{}^{@}$ will increase in size as future contingent events become present and it will become saturated as the present reaches time t.

Let us turn to our case and consider the semifactuals (SFi) to (SFiv). We will name the relevant events: ¬A: Johann's inviting Bettina; e: Ludwig's coin coming up heads; C: Bettina's being at the party. The most refined time series that needs to be considered in this case is $\langle t_{\neg A}, t_e, t_C\rangle$. The time of utterance of these semifactuals in question is at $t_{\neg A}$ (for (SFi)), after $t_{\neg A}$ but before t_e (for (SFii)), after t_e but before t_C (for (SFiii)) and after t_C (for (SFiv)). For each semifactual, $\mathcal{E}_{t_{\neg A}}{}^{@}$ contains Johann's inviting Bettina and $\mathcal{E}_{t_{\neg A}}{}^{A}$ contains Johann's not inviting Bettina.

First, consider (SFi) and (SFii). **Before $\mathbf{t_e}$**, $\mathcal{E}_{t_e}{}^{@}$ does not contain e, since e is a future contingent event. Neither does $\mathcal{E}_{t_e}{}^{A}$, since e is nonexistent as an actual event and, considering its status as a possible event, it is not entailed by ($\mathcal{E}_{t_{\neg A}}{}^{A}$&L). $\mathcal{E}_{t_C}{}^{@}$ does contain C, since C is a future necessary event: C is entailed by ¬A in conjunction with the laws of nature and true statements of fact about Bettina's disposition. Does $\mathcal{E}_{t_C}{}^{A}$ contain C? $P(C|\mathcal{E}_{t_e}{}^{A}) = 1/2$, since C's obtaining is dependent on a coin toss. But $P(C|\mathcal{E}_{t_e}{}^{@}) = 1$, since $\mathcal{E}_{t_e}{}^{@}$ contains ¬A and C is entailed by ¬A in conjunction with the laws of nature and true statements of fact about Bettina's disposition. Hence, C is not contained in $\mathcal{E}_{t_C}{}^{A}$, since, considering C's status as an actual event, $P(C|\mathcal{E}_{t_e}{}^{@}) > P(C|\mathcal{E}_{t_e}{}^{A})$ and, considering C's status as a possible event, it is not entailed by ($\mathcal{E}_{t_e}{}^{A}$&L). Consequently, (SFi) and (SFii) come out false on my analysis, which matches the desired truth-values.

Now let us consider (SFiii) and (SFiv). **After $\mathbf{t_e}$**, $\mathcal{E}_{t_e}{}^{@}$ contains e since e has temporally become. $\mathcal{E}_{t_e}{}^{A}$ does contain e as well, since, considering e's status as an actual event, $P(e|\mathcal{E}_{t_{\neg A}}{}^{A}) = 1/2 = P(e|\mathcal{E}_{t_{\neg A}}{}^{@})$. $\mathcal{E}_{t_C}{}^{@}$ contains C, since, during the open time interval (t_e, t_C), C is a future necessary event and, from t_C on, C has temporally become. $\mathcal{E}_{t_C}{}^{A}$ contains C, since, considering its status as an actual event, $P(C|\mathcal{E}_{t_e}{}^{A}) = 1 = P(C|\mathcal{E}_{t_e}{}^{@})$. Consequently, (SFiii) and (SFiv) come out true on my analysis, which matches the desired truth-values.

c. *Peirce vs. Broad on the Ontic Status of Future Events.* Peirce believes that future contingent events and only future contingent events come into existence as time progresses. But Peirce's view is somewhat of a minority dynamic view of time. On the standard dynamic view of time, all future events (and not just future contingent events) come into existence as time progresses. For instance, in *Scientific Thought* Broad writes:

> . . . when we say that the red section [of the history of a signal lamp] precedes the green section, we mean that there was a moment when the sum total of existence included the red event and did not include the green one, and that there was another moment at which the sum total of existence included all that was included at the first moment and also the green event. (. . .) when an event becomes, it *comes into existence*; and it was not anything at all until it had become. You cannot say that a future event is one that succeeds the present; for a present event is one that is succeeded by nothing. (1927: 67–68)

It is easy to show that my theory of counterfactuals yields the correct truth values on this standard dynamic view of time as well. Peirce and Broad disagree on the status of future necessary events. Suppose that at time ti, e is a future necessary event in the half-open time interval (ti, t(i+1)] – i.e. $P(e|\mathcal{E}_{ti}{}^{@}) = 1$. For Peirce, e is an existent actual event at time ti. Since what is actual is also possible, e will be included in $\mathcal{E}_{t(i+1)}{}^{A}$, if and only if, considering its status as an actual event, $P(e|\mathcal{E}_{ti}{}^{@}) = 1 = P(e|\mathcal{E}_{ti}{}^{A})$ or, considering its status as a possible event, it is entailed by $(\mathcal{E}_{ti}{}^{A}\&L)$ – i.e. $P(e|\mathcal{E}_{ti}{}^{A}) = 1$. For Broad, e is not an existent actual event at time ti. Hence, e will be included in $\mathcal{E}_{t(i+1)}{}^{A}$, if and only if, considering its status as a possible event, it is entailed by $(\mathcal{E}_{ti}{}^{A}\&L)$ – i.e. $P(e|\mathcal{E}_{ti}{}^{A}) = 1$. Hence, on either view, the necessary and sufficient condition for the inclusion of a future necessary event e at ti (i.e. an event such that $P(e|\mathcal{E}_{ti}{}^{@}) = 1$) in $\mathcal{E}_{t(i+1)}{}^{A}$ is precisely the same – viz. that $P(e|\mathcal{E}_{ti}{}^{A}) = 1$. Whether Peirce's view or Broad's view is adopted does not make any difference to the construction of $\mathcal{E}_{t_C}{}^{A}$ and hence to the truth values of the relevant semifactuals.

VI. CONCLUSION

It is a puzzling feature about sequential counterfactuals that they bear time-relative truth-values. I set off in Goodman's footsteps to account for this feature. There is some confusion in the literature about what constitutes Goodman's cotenability account. I defended, against Goodman, the positive semifactual test over the negative counterfactual test of cotenability. To break out of Goodman's circular account of counterfactuals, a probabilistic theory of semifactuals was constructed. Having put all of these components in place, I argued that a combination of (i) Goodman's cotenability account, substituting the positive semifactual test for the negative counterfactual test, (ii) a probabilistic theory of semifactuals and (iii) a dynamic view of time can account for the fact that sequential counterfactuals bear time-relative truth-values.

NOTES

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[1] Lewis (1973: 2) defends the following equivalence between *would*-counterfactuals and *might*-counterfactuals: A $\Diamond\!\!\rightarrow$ not-C (i.e. if A were the case, then C might not be the case) if and only if not-(A $\Box\!\!\rightarrow$ C) (i.e. it is not the case that if A were the case, then C would be the case). The principle is not uncontroversial (e.g. see Stalnaker, 1980: 98), but I take the particular applications of the principle in this and later inferences to be intuitively acceptable.

[2] I let $\Box\!\!\rightarrow$ take on pairs of non-tensed propositions as its arguments. One might object that $\Box\!\!\rightarrow$ takes on pairs of tensed rather than non-tensed propositions as its arguments. But my point still holds: the proposition expressed by

(CFtensed) not-Invite$_{\text{past}}$(Johann, Bettina, t) $\Box\!\!\rightarrow$ Bestow$_{\text{future}}$ (Bettina, Johann, t')

takes on different truth-values as it is expressed before Ludwig's tossing his coin (i.e. by CFii) as opposed to after Ludwig's tossing his coin (i.e. by CFiii).

[3] I am assuming here that the conditions for the additivity of voltage are satisfied.

[4] It is an issue of contention whether the *even-if* semifactual entails the *if*-semifactual, but I take it to be uncontroversial that the *if*-semifactual entails the *even-if* semifactual. To see this, note that in the case Johann's intention to come to the party exerts a positive influence on Bettina's intention to come to the party, it would not be incorrect to say: 'Even if Johann were to come to the party, Bettina would come. In fact, if Johann were to come, Bettina would most certainly come.'

[5] Over the last decade, Kvart has tried twice to construct a probabilistic analysis of semifactuals and I wish to acknowledge my debts here to his seminal work. The earlier view is to be found in Kvart (1986, 1992) and is abandoned in Kvart (1991a). The later view is developed in Kvart (1991b, 1994). A discussion of Kvart's work is beyond the scope of this paper.

[6] I will restrict my attention to sequential counterfactuals containing momentary and stochastic antecedent events. Such events can occur while all other events prior to or at the time of their occurrence remain fixed, respecting the laws of nature. I expect that the theory can be extended to cover sequential counterfactuals with antecedent events that are non-momentary and that are incompatible with prior events through the laws of nature, but a whole new range of intuitions will need to be tapped to do so.

[7] Actual events are also possible events. Hence, strictly following our directions for the construction of sets of counterfactual events, both the criteria for the inclusion of actual and possible events need to be evaluated. However, if some actual event is not included considering its status as an actual event, then it follows that it will not be included considering its status as a possible event either. To see this, consider some actual event e during (ti,t(i+1)]: if, considering its status as an actual event, e is not included in $\mathcal{E}_{t(i+1)}{}^{A}$, then it must be the case that $P(e|\mathcal{E}_{ti}{}^{A})$ < 1 and, hence, considering its status as a possible event, e cannot be entailed by $(\mathcal{E}_{ti}{}^{A}\&L)$ and so cannot be included in $\mathcal{E}_{t(i+1)}{}^{A}$.

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