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Voting Procedures for Complex Collective Decisions. An Epistemic Perspective

LUC BOVENS AND WLODEK RABINOWICZ*

Abstract. In addressing a complex issue that is decomposable into several subquestions, a committee can use different voting procedures: Either it can let the committee members vote on each sub-question and then use the outcomes as premises for its conclusion on the main issue (*premise based-procedure*, *pbp*), or it can let the members directly vote on the conclusion (*conclusion-based procedure*, *cbp*). The procedures can lead to different results, but which of them is a better truth-tracker? On the basis of Condorcet's jury theorem, we show that the pbp is clearly superior if the objective is to reach truth for the right (= correct) reasons. However, if the goal instead is to reach truth for whatever reasons, right or wrong, there will be cases in which using the cbp turns out to be more reliable, even though, for the most part, the pbp will retain its superiority. In that connection, we also consider the truth-tracking potential of a "sophisticated" variant of the pbp, which is sensitive to the size of the majorities supporting each of the premises.

1. The Problem

A committee faces a complex question that divides into several subquestions. As an illustration, consider academic tenure appointments. Both the teaching skills and the research skills of the candidates are usually deemed to be relevant and the candidates are required to meet some standard on both skills. The dean's decision on tenure is informed by a faculty

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vote in the home department of the candidate. Here is a question about procedure. The dean might ask each faculty member to assess the candidate on teaching and research and to cast a "yes" vote for tenure if and only if she deems the candidate to be worthy on both. Tenure will be granted only if there is sufficient support for the candidate. Call this the *conclusion-based procedure* (cbp). Or the dean might ask each faculty member to cast a vote on whether the candidate is worthy on teaching and another vote on whether the candidate is worthy on research. Tenure will be granted only if there is sufficient support for the candidate both on teaching and research. Call this the *premise-based procedure* (pbp). That the two procedures are not equivalent has been noted in legal theory in connection with jury votes (cf. Kornhauser and Sager 1986, 1993; Kornhauser 1992a, 1992b; Chapman 1998a, 1998b). There may be a majority of voters supporting each premise, but if the overlap between these majorities is limited, there will be a majority against the conclusion. This is a common procedural problem in democratic decision-making when the decision to be made concerns a complex issue that can be broken down into several sub-questions.

Pettit (2001) connects the choice between the two procedures with general issues in political theory, in particular with the discussion of *deliberative democracy*.[1] Deliberative democracy calls for a public process of deliberation and reasoning. An important element of that political ideal is a *contestability* requirement on democratic regimes. It should be possible for citizens to challenge democratic decisions by questioning their underlying reasons. The pbp makes such contestability much easier. It gives the premises of an argument a democratic imprimatur and thus places them in the public arena. It thereby allows for contestation of the conclusion by questioning its premises. The cbp, on the other hand, keeps the premises out of the public arena, which works against making the democratic regime accountable for the reasons behind its decisions.

There is a more direct connection between deliberative democracy and the pbp that is not mentioned by Pettit. Deliberative democracy calls for a collective process of reasoning. The pbp clearly involves such a process; in this procedure, the group as a whole takes a stand on the premises and then draws the conclusion. By contrast, in the cbp, the reasoning from premises to conclusion is conducted by each individual separately.

Here, however, the problem we want to examine concerns the relative advantages and disadvantages of the two procedures from the *epistemic* point of view. In some cases one can assume that the question before the group has a *right answer*, which the group is trying to reach. How do the two procedures compare in their truth-tracking capacities?

It need not always be the case that there is an independent truth to be reached, which can be tracked by a democratic voting procedure. In some

[1] This connection is also made in Brennan 2001. For a general characterization of deliberative democracy, cf. Elster 1998.

contexts, the right decision is simply the decision that is reached by a legitimate political procedure. Still, it seems that such a purely *procedural* reading of right and wrong would often be inappropriate. For example, in many cases, the voters on the losing side might well consider the majority decision to be wrong, even if they are prepared to abide by it. What they object to is not the legitimacy of the decision-making process but its outcome. And the objections need not be framed in terms of their personal interests; they might well appeal to the goals of the collective or to other impersonal criteria. The minority voters might argue that the decision, however legitimate, was incorrect. If such arguments make sense, then it is meaningful to evaluate collective decision-making procedures from the epistemic perspective and compare their capacities as truth-trackers. In fact, *epistemic democrats* take democracy to be especially valuable from a truth-oriented perspective (cf. Estlund 1990, 1993, 1997, 1998; List and Goodin forthcoming).[2] Rousseau is often seen as a founding father of this approach to democracy. It is central in his theory that voters express their views on the "general will" rather than their individual preferences (cf. Rousseau 1997, book 4, chap. 2). In a modified version, this idea is retained by deliberative democrats, who require voters to express their opinions as to which decision is best from the point of view of the common goals of the collective. Another French Enlightenment figure, Marquis de Condorcet, is given credit for the theorem that is meant to clarify democracy's epistemic advantage.[3] The Condorcet Jury Theorem will be our point of departure in the comparison of the two procedures.

So which of the two procedures is a better truth-tracker? As will be shown, the answer to this question is by no means univocal. Starting from Condorcet's Jury Theorem, we will identify the features of the decision-making situation that determine whether and in what respect the pbp or the cbp is better at tracking truth. We will show that the pbp is clearly superior if we want to reach truth for the *right reasons*, i.e., without making any mistakes on the road to the conclusion. However, if the goal is to reach truth for *whatever reasons*, right or wrong, there will be a range of cases in which using the cbp turns out to be more reliable. But, for the most part, the pbp will retain its superiority.[4]

2. The Model

We start with a simple model that is based on the Condorcet Jury Theorem. The theorem itself, in one of its versions, can be stated as follows:

[2] The label itself, "an epistemic theory of democracy," comes from Cohen 1986.
[3] Cf. Condorcet 1785, pp. 279ff.; for an English translation of the relevant passages, see McLean and Urken 1995.
[4] Our results partly confirm and partly disconfirm the conjectures made in Pettit and Rabinowicz 2001.

Suppose a group of n voters, where n is odd and greater than one, have to assess a proposition A. Suppose that for some p such that $1 > p > .5$, each voter has a chance p of correctly assessing whether A is true or not[5] and that this chance is independent of whether the other voters' assessments are correct or not. Then the probability that the majority vote is a correct assessment of whether A is true or not is greater than p and converges to 1 as the number of voters increases to infinity.

To apply the theorem in the context of voting on tenure we need to make some idealizations that are more or less realistic depending on the situation at hand:

(i) The number of voters is odd;
(ii) For some p such that $1 > p > .5$, each voter has the same chance p of making a correct assessment of worthiness on teaching and research, respectively;
(iii) The chance of a correct assessment for a given voter does not depend on whether other voters' assessments are correct.

Some of these idealizations may be relaxed, but at the price of greater complexity. Concerning assumption (i), it can be shown that for all even $n > 2$, there exists a number $p(n) \in (.5, 1)$, such that the theorem holds for any $p \in (p(n), 1)$. Furthermore, $p(n)$ is a decreasing function of the even numbers n and approaches .5 as n approaches infinity. Concerning assumption (ii), the requirement that a voter's level of competence is the same for all voters can be relaxed as long as their *average* competence of correctly assessing whether the proposition is true or not is contained in (.5, 1).[6] As for assumption (iii), the requirement that voters cast their votes independently can also be relaxed. In particular, even if the voters are to some extent influenced by common opinion leaders, the majority is still more trustworthy than a single voter. The Condorcet Jury Theorem applies as long as the influence of opinion leaders is not too overwhelming (cf. Estlund 1994). In what follows, however, we shall ignore these complications.

Essentially, the idea behind the Condorcet Jury Theorem is simple. If the competence of each voter is independent of other voters' being correct in their assessments, they may be treated as independent witnesses. If independent witnesses are reasonably competent but none of them is infallible and if we have to make up our minds one way or the other, then consulting more witnesses rather than fewer, and going by what the most witnesses tell us, is always advisable.

To introduce our methodology, let us construct a function that measures the probability that the majority vote provides a correct assessment of a

[5] Each voter votes either for or against A; it is assumed that there are no abstainers. The value p is the probability that the voter votes for A if and only if A is true. Since there are no abstainers, p is also the probability the voter votes against A if and only if A is false.
[6] Cf. Borland 1989; Grofman, Owen and Feld 1983, Owen, Grofman and Feld 1989.

certain proposition for different values of $p \in (.5, 1)$ and for different odd numbers n of voters. We number the voters from 1 to n. The probability that the first k voters are correct and the remaining $n - k$ voters are incorrect is $p^k(1 - p)^{(n-k)}$. There are $\binom{n}{k}$ ways to pick out k individuals out of a group of n voters. Hence, the probability that precisely k out of n voters are correct is:

$$\binom{n}{k} p^k (1-p)^{(n-k)} \tag{1}$$

For k voters to be a majority among n voters—among n voters, if n is odd, k has to be at least as large as $(n + 1)/2$. Now, let M be the proposition that a majority among n voters is correct. The probability that M holds (for an odd n) is:

$$P(\mathrm{M}) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k (1-p)^{(n-k)} \tag{2}$$

In figure 1 we have plotted this function for p ranging from 0 to 1 and for $n = 3, 11, 101$. For any p above .5 and below 1, (i) $P(\mathrm{M})$ is always greater than p, and (ii) the greater the number of voters, the more confident we may be that the majority gets it right.

So far, we have just considered voting on a single proposition. Let us now turn to the more complex case of a tenure vote. Consider the following three propositions:

(P) The candidate is worthy of tenure on teaching.

(Q) The candidate is worthy of tenure on research.

(R) The candidate is worthy of tenure *tout court* (i.e., both P and Q are the case).

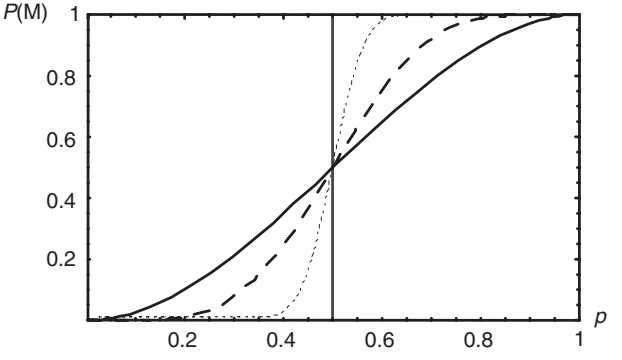

Figure 1: The chance that the majority is correct for different levels p of voter competence and for $n = 3$ (full line), 11 (dashed line) and 101 (dotted line) voters.

Let us first model the pbp. Suppose the dean asks the faculty to vote on P and to vote on Q and will grant tenure if and only if a majority casts a positive vote on P and a majority casts a positive vote on Q. We need to distinguish between four different *situations*, as regards the actual truth-values of the propositions P and Q:

(S1) P & Q
(S2) not-P & Q
(S3) P & not-Q
(S4) not-P and not-Q.

The candidate is worthy of tenure *tout court* in situation S1, but not in situations S2 to S4.

Institutions tend to have different standards: The chance that some arbitrary candidate makes the mark on teaching or makes it on research will be very different from one school to another. Now let us make some additional simplifying assumptions, which again can be relaxed if needed:

(iv) The *ex ante* chance that an arbitrary candidate in a particular school is worthy of tenure on teaching, considering the standards that are upheld within the school, is the same as the *ex ante* chance that the candidate is worthy of tenure on research, i.e., $P(\mathrm{P}) = P(\mathrm{Q}) = q$, where $0 < q < 1$;
(v) As regards these chances, P and Q are mutually independent;
(vi) The chance of a correct assessment of a factor by a given voter is the same in every situation and is independent of whether his or any of the other voters' assessments of the other factor are correct or not.

The chance that the pbp will yield a correct assessment of whether the candidate is worthy of tenure is:

$$P(\mathrm{M}^{\mathrm{pbp}}) = \sum_{i=1}^{4} P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{S}i)P(Si) \tag{3}$$

Given the simmetry assumption (iv) and the independence assumption (v), it is easy to determine $P(\mathrm{S}i)$: $P(\mathrm{S1}) = q^2$; $P(\mathrm{S2}) = P(\mathrm{S3}) = q(1 - q)$; $P(\mathrm{S4}) = (1 - q)^2$. To determine the conditional probabilities $P(\mathrm{M}^{\mathrm{pbp}} \mid \mathrm{S}i)$, we need to assess each situation individ-ually: Let us consider situation S2 (in which the candidate is worthy on research but not on teaching) to illustrate the reasoning. In S2, the candidate in question is not worthy of tenure. There are three mutually exclusive ways in which the dean can reach this correct decision: (i) the majority is right in their assessment of the candidate's teaching and is right in their assessment of the candidate's research (note that the two majorities need not coincide); (ii) the majority is right in their assessment of the candidate's teaching, but wrong in their assessment of the candidate's research; (iii) the majority is wrong in their assessment of the candidate's

teaching and in their assessment of the candidate's research. Hence, given the independence in the assessments of teaching and research:

$$P(\mathrm{M}^{\mathrm{pbp}}|\mathrm{S2}) = P(\mathrm{M})^2 + P(\mathrm{M})(1-P(\mathrm{M})) + (1-P(\mathrm{M}))^2 \tag{4}$$

Note that in case (i) the right decision is reached for the right reasons, while in cases (ii) and (iii) the right decision is reached for the wrong reasons.

Equation (3) provides the chance that the pbp yields a correct assessment, whether for the right or for the wrong reasons, i.e., *for whatever reason*. But one might also want to know the chance of the pbp delivering a correct assessment *for the right reasons* (rr) only. It is easy to see that, in each of the four situations, this chance is $P(\mathrm{M})^2$. Hence,

$$P(\mathrm{M}^{\mathrm{pbp-rr}}) = P(\mathrm{M})^2 \tag{5}$$

We turn to the cbp. For any given situation S*i*, the chance that the cbp will yield the correct assessment of whether the candidate is worthy of tenure is given by

$$P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{S}i) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} P(\mathrm{V}|\mathrm{S}i)^k (1-P(\mathrm{V}|\mathrm{S}i))^{(n-k)} \tag{6}$$

where $P(\mathrm{V}\,|\,\mathrm{S}i)$ is the chance, given S*i*, that a particular voter casts a correct vote on tenure *tout court*. Consequently, the unconditional chance that the cbp will yield the correct assessment is:

$$P(\mathrm{M}^{\mathrm{cbp}}) = \sum_{i=1}^{4} P(\mathrm{M}^{\mathrm{cbp}}|\mathrm{S}i)P(\mathrm{S}i) \tag{7}$$

To illustrate how we go about calculating $P(\mathrm{V}\,|\,\mathrm{S}i)$, we once again focus on situation S2. In S2, to cast a correct vote on tenure *tout court*, the voter has to be either (i) correct on both teaching and research, (ii) correct on teaching, but not on research, or (iii) wrong on both teaching and research:

$$P(\mathrm{V}|\mathrm{S2}) = p^2 + p(1-p) + (1-p)^2 \tag{8}$$

Note that in case (i), the voter reaches the right decision for the right reasons, while in cases (ii) and (iii) he comes to the right decision for the wrong reasons.

(8) provides the chance that the cbp yields a correct assessment, whether for the right or for the wrong reasons, i.e., *for whatever reason*. But once again, one might want to know the chance of the cbp delivering a correct assessment *for the right reasons* (rr) only, i.e., the chance of a majority among the voters making a right assessment of the conclusion for the right reasons. In

each of the four situations, the chance that a particular voter casts a correct vote for the right reasons is p^2. Hence,

$$P(\mathrm{M}^{\mathrm{cbp-rr}}) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} (p^2)^k (1-p^2)^{(n-k)} \tag{9}$$

We can now compare the two procedures for their truth-tracking potential: Which procedure is more likely to provide a correct assessment of the candidate in question?

3. Sample Results

Concerning the capacity of the two procedures as regards truth-tracking *for the right reasons*, the pbp is superior. To see this, note that the pbp yields a correct assessment of the conclusion based on the right reasons whenever (i) there is a majority that correctly assesses one premise and also (ii) a majority that correctly assesses the other premise. The cbp, on the other hand, correctly assesses the conclusion for the right reasons if and only if (iii) there is a majority that correctly assesses *both* premises. Obviously, (iii) entails (i) and (ii), but not vice versa. Therefore, whenever the cbp makes a right assessment for the right reasons, the pbp would make it as well. But there are possible cases in which the pbp would make a right assessment for the right reasons but the cbp would fail. Such cases (in which (i) and (ii) hold, but (iii) does not) have a non-zero probability as long as the voters' competence with respect to one premise is at least partly independent of the correctness of their assessments regarding the other premise. And we have assumed full independence for this case (cf. assumption (vi)).

Still, it is one thing to know that pbp is superior as a truth-tracker for the right reasons and another to find out the extent of this superiority. In figure 2, we provide the answer to this question by comparing $P(\mathrm{M}^{\mathrm{pbp-rr}})$ with $P(\mathrm{M}^{\mathrm{cbp-rr}})$, for various numbers of voters and for various values of p.

As can be seen, the superiority of the pbp is more *pronounced* for larger numbers of voters but the *p-interval* over which that superiority can be observed decreases as the number of voters increases. For n approaching infinity, Δ goes to 0 for all p in the intervals [0, .5] and $[\sqrt{.5}, 1]$, and it goes to 1 for all $p \in (.5, \sqrt{.5})$. For lower n, this curve is relatively flat, has a broad base and its maximum is close to $p = \sqrt{.5}$, while as n grows the curve becomes steeper, its base becomes smaller and its maximum recedes towards $(.5 + \sqrt{.5})/2$.

Suppose, on the other hand, that we are not concerned about the reasons that went into the deliberation: We are interested in the chances that the procedures yield the right results, whether these are arrived at for the right or for the wrong reasons. Now, the cbp does better than the pbp when it comes to reaching truth for the wrong reasons. The explanation is quite simple.

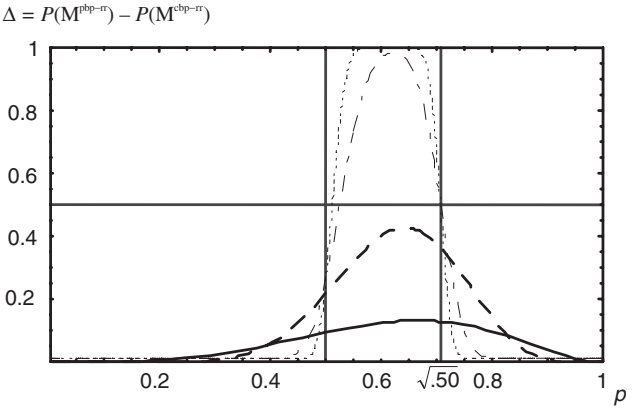

Figure 2: The difference between the chances that the majority vote is correct for the right reasons for the pbp and for the cbp, for $n = 3$ (full line), 11 (dashed line), 101 (dot-dashed line) and 501 (dotted line).

Truth can be reached for the wrong reasons only when P&Q is false. In such situations, however, the cbp will yield the right conclusion whenever the pbp does, but it will also yield the right conclusion in some cases when the pbp goes wrong: There may be a majority against the conclusion even if there is a majority for each premise. Consequently, the pbp should lose much of its superiority when we assess the capacities of the procedures to track truth for *whatever* reasons. In figure 3, we compare the two procedures in this respect, for various numbers of voters and for various values of p and q.

For the most part, pbp is better at tracking truth for whatever reasons. However, the cbp tends to do better as we decrease the competence of the voters (low p), the size of the committee (low n), and the leniency of the context (low q). The reasons are as follows. (i) The cbp does better than the pbp when it comes to reaching truth by mistake, i.e., for the *wrong* reasons, which can only happen when the conjunction is false. Individual voters are more prone to commit mistakes if they are less competent; a majority of the voters is more prone to commit a mistake if there are few of them; and the room for truth by mistake is larger in less lenient contexts, in which the probability of the conjunction being false is higher. (ii) As the number of voters increases, the pbp does increasingly better than the cbp with less competent voters, when it comes to reaching truth for the *right* reasons. But for small numbers of voters, this advantage of the premise-based procedure is smaller and thus it cannot outweigh that procedure's disadvantages as regards reaching truth by mistake.

We conclude that, in *some* contexts (low p, low q, low n), the cbp does better than the pbp. Epistemic democrats may come to favor the cbp over the pbp in these contexts. Deliberative democrats, on the other hand, might

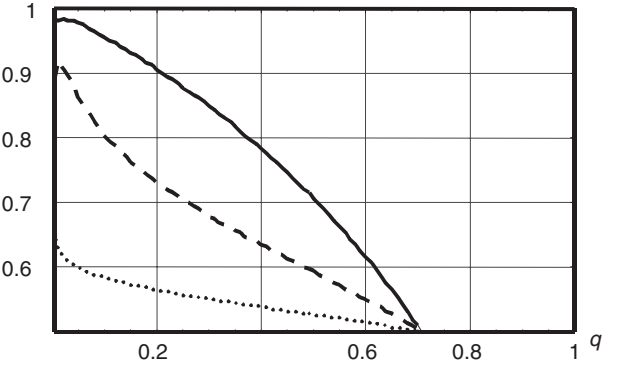

Figure 3: Phase curves for $n = 3$ (full line), 11 (dashed line), 101 (dotted line), for p between .5 and 1, and for q between 0 and 1. Underneath the phase curve, the cbp is a better truth-tracker; above the phase curve, the pbp is better.

be argued to be less interested in arriving at truth, by whatever means, than in reaching collective knowledge (cf. Pettit 2001). On the classical analysis of that concept, knowledge is true belief supported by the right reasons. The pbp always does better than the cbp in tracking truth *for the right reasons*. Hence, there is a tension between deliberative democracy and epistemic democracy. In some contexts, epistemic democrats will come to favor the cbp over the pbp, whereas deliberative democrats will always favor the pbp over the cbp.[7] In the next section, we will show how this tension can be at least partially resolved by shifting from the pbp to a more sophisticated variant of the premise-based procedure.

4. The Sophisticated Premise-Based Procedure

The pbp, as it stands, is insensitive to the percentages of voters that support each premise. Even when the majorities behind the premises are very weak, the pbp will tell us to accept the conjunction. A *sophisticated* premise-based procedure (spbp, for short) would give weight to the strength of majorities. Here is how this can be done: We can think of the percentage of voters that

[7] However, as was pointed to us by Peter Pagin, even deliberative democrats, who put a special premium on collective knowledge, should not take lightly the advantages of the cbp as a truth-tracker. The classical conception of knowledge as true belief based on right reasons is not uncontested. On the competing *reliabilist* view, knowledge instead is defined as true belief arrived at by reliable methods (cf. Goldman 1979). From this reliabilist perspective, therefore, the cbp may sometimes be better than the pbp as a generator of collective knowledge. This will be the case in all those contexts in which the former procedure is more reliable in arriving at truth. Furthermore, as pointed out to us by Thomas Schmidt, even if it is collective knowledge in the *classical* sense we are after—a true belief for the right reasons—the cbp may still be preferable in the contexts in which that procedure is more reliable. It is enough that the group *knows* that the context is of this kind. For if the group knows this, it thereby has a *right reason* to accept the verdict delivered by that procedure: a right reason of a higher order, so to speak.

support a proposition A as a measure of the strength of a collective belief. If k out of n voters vote yes to A, then the proposition is collectively believed with strength k/n. If the conjuncts in a conjunction are mutually independent, the sophisticated premise-based procedure makes the acceptance of the conjunction dependent on the value of the *product* of the strengths of collective beliefs in each conjunct. If that product exceeds .5, the spbp tells us to accept the conjunction; if it is lower than .5, we should reject it. Thus, the conjunction will *not* be accepted if the majorities behind each premise are relatively weak. Consequently, the pbp and the spbp are not equivalent. The latter procedure imposes stricter requirements on acceptance than the former. Nor is the spbp equivalent to the cbp. If a weak majority supports the conjunction but everybody else rejects both conjuncts, then the cbp will tell us to accept the conjunction while the spbp will reject it. On the other hand, if the majority behind each conjunct is, say, less than $^3/_4$ and greater than $\sqrt{.5}$, but the overlap between these majorities is minimal (and hence less than one half of the total number of voters), then the spbp will tell us to accept the conclusion but the cbp will reject it.

How does the spbp manage in its capacity as a truth-tracker (for whatever reasons, right or wrong)? Let us first assume that both premises P and Q are true. Let j be the number of voters voting for P and k the number of voters voting for Q. Then the spbp will yield the correct result just in case

$$j/n \times k/n > .5 \tag{10}$$

Clearly, the lower bound for j that permits the *possibility* of the conclusion being accepted is $(n + 1)/2$. The upper bound for j is n. From (10), it follows that for each number j in this interval, k must satisfy the following inequality for the conclusion to be accepted:

$$k > \frac{.5 \times n^2}{j} \tag{11}$$

Let $C(x)$ be the *ceiling* function which, for every real number x, yields the smallest integer greater than or equal to x. Then the chance that the spbp will yield the correct result in situation S1, i.e., when P and Q are both true equals:

$$P(\mathrm{M}^{\mathrm{spbp}}|\mathrm{S1}) = \sum_{j=\frac{n+1}{2}}^{n} \left(\binom{n}{j} p^j (1-p)^{n-j} \sum_{k=C\left(\frac{.5n^2}{j}\right)}^{n} \binom{n}{k} p^k (1-p)^{n-k} \right) \tag{12}$$

For S2, i.e., when P is true and Q is false, the chance that the spbp will yield the wrong result is the chance that the product of the proportion of voters

who get it right on P and the proportion of voters who get it wrong on Q is greater than .5, so that the conclusion is accepted. The chance for an individual voter to get it wrong on Q is $1 - p$. Hence, the chance that the spbp will yield the right result in situation S2 equals

$$P(\mathrm{M}^{\mathrm{spbp}}|\mathrm{S2}) = 1 - \sum_{j=\frac{n+1}{2}}^{n} \left(\binom{n}{j} p^j (1-p)^{n-j} \sum_{k=\mathrm{C}\left(\frac{.5n^2}{j}\right)}^{n} \binom{n}{k} (1-p)^k p^{n-k} \right) \tag{13}$$

Clearly, $P(\mathrm{M}^{\mathrm{spbp}} \mid \mathrm{S2}) = P(\mathrm{M}^{\mathrm{spbp}} \mid \mathrm{S3})$. By a similar argument,

$$P(\mathrm{M}^{\mathrm{spbp}}|\mathrm{S4}) = 1 - \sum_{j=\frac{n+1}{2}}^{n} \left(\binom{n}{j} (1-p)^j p^{n-j} \sum_{k=\mathrm{C}\left(\frac{.5n^2}{j}\right)}^{n} \binom{n}{k} (1-p)^k p^{n-k} \right) \tag{14}$$

Hence, the chance that the spbp will yield a correct assessment of whether the candidate is worthy of tenure is:

$$P(\mathrm{M}^{\mathrm{spbp}}) = \sum_{i=1}^{4} P(\mathrm{M}^{\mathrm{spbp}}|\mathrm{S}i) P(\mathrm{S}i) \tag{15}$$

We have plotted which procedure is the better truth-tracker for various sizes of committees n, various competencies p and various *ex ante* chances q that the candidate is qualified for tenure on at least one of the criteria. For relatively small committees, either the pbp or the spbp is the better truth-tracker, as we can read off from figures 4 and 5. This is welcome news for deliberative democrats, for it means that at least for relatively small committees, there always exists *some* premise-based procedure (the pbp or the spbp) that is a better truth-tracker than the conclusion-based procedure. But for larger committees,[8] there are some values of p and q for which the cbp is the better truth-tracker, as we can read off from figures 6 and 7.

It is easy to see why the spbp has at least a chance of taking over from the cbp. The cbp gains its advantage over the pbp because it is better at rejecting unqualified candidates. It is better at doing so because it exploits the possibilities of rejecting unqualified candidates for the wrong reasons. On the spbp, it becomes more difficult to accept a candidate than on the pbp, since we need more than just simple majorities before the candidate is accepted. Hence, the spbp, just like the cbp, more readily rejects candidates.

[8] The smallest committee in which the cbp outperforms both the pbp and the spbp for some values of p and q is a committee of 11 voters. We chose to present the results for 13 voters, since the region in which the cbp outperforms the pbp and spbp is so small for 11 voters that it is barely visible in a large-scale graph.

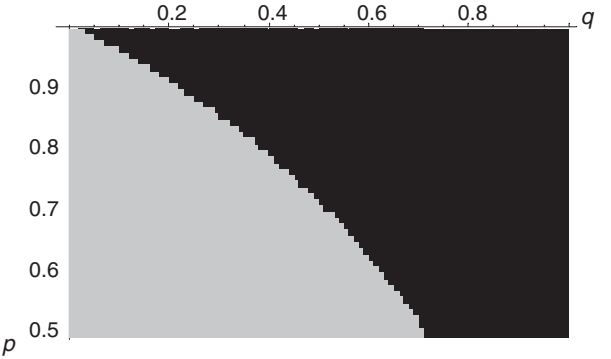

Figure 4: A comparison between pbp, spbp and cbp for 3 voters. The pbp is the best truth-tracker in the black area and the spbp is the best truth-tracker in the grey area.

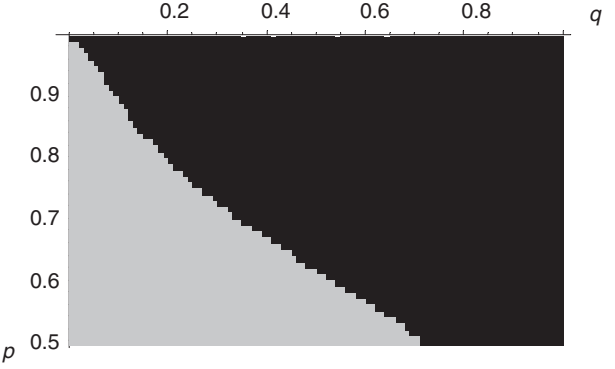

Figure 5: A comparison between pbp, spbp and cbp for 9 voters. The pbp is the best truth-tracker in the black area and the spbp is the best truth tracker in the grey area.

And that is where it gains its advantages over the pbp: Just like the cbp, it exploits the possibilities of rejecting unqualified candidates for the wrong reasons. We do not have an intuitive explanation of when the spbp is capable of taking over from cbp and when it is not. This issue remains to be examined.

5. Further Questions

So far we have laid out the basic model and indicated a couple of fairly striking results. The basic model makes a range of assumptions that require careful scrutiny. Here are a number of open questions that need to be addressed:

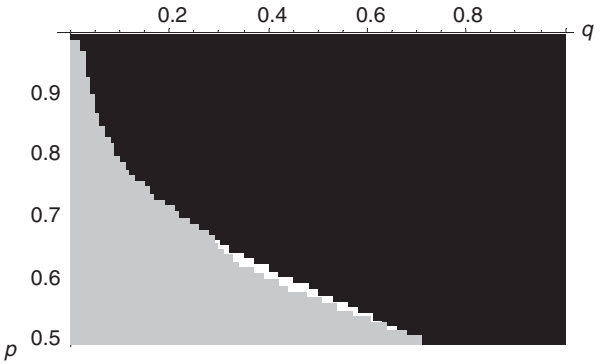

Figure 6: A comparison between pbp, spbp and cbp for 13 voters. The pbp is the best truth-tracker in the black area, and the spbp is the best truth tracker in the grey area. In the white area, the cbp is the best truth-tracker.

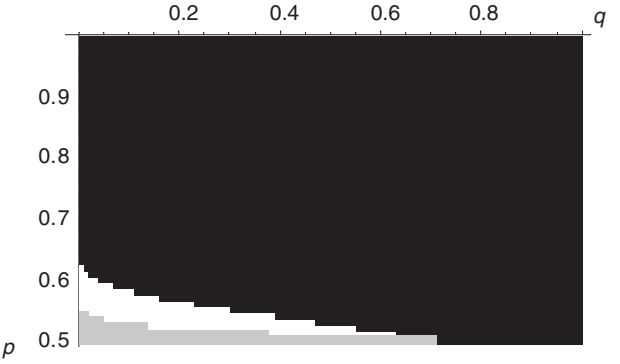

Figure 7: A comparison between pbp, spbp and cbp for 101 voters. The pbp is the best truth-tracker in the black area, and the spbp is the best truth tracker in the grey area. In the white area, the cbp is the best truth-tracker.

(i) How do the procedures perform when the number of voters increases to infinity[9]?
(ii) What happens when we increase the number of propositions in the conjunction?
(iii) What happens if we relax our various symmetry and independence assumptions?
(iv) The cbp keeps the reasoning of the voters hidden while the premise-based procedures (the pbp and the spbp) bring that reasoning into the open. So do these procedures gain an epistemic advantage when

[9] For an excellent discussion of this issue, see List forthcoming.

the voters cannot be trusted to base their decisions on the prescribed criteria, e.g., when they may be focusing only on research and ignoring teaching, or when they may be adding various private concerns as additional criteria in their decision-making?

(v) The premise-based procedures tempts one to vote *strategically*: to vote against a premise one accepts, if one rejects the whole conjunction but suspects that others might accept both conjuncts. Strategic voting is impossible with the cbp. How much of an epistemic advantage does this give to the latter procedure?

(vi) How is the choice of procedure affected by taking into consideration the respective *utilities* and *disutilities* of accepting a tenure-worthy candidate, accepting a non-tenure-worthy candidate, rejecting a tenure-worthy candidate and rejecting a non-tenure-worthy candidate? In many situations, there is a utility-based asymmetry between the acceptance of the conclusion and its rejection, not least in legal contexts: A wrong verdict of guilt is more devastating that a wrong verdict of innocence.

(vii) We have only considered a case in which a group takes a stand on a conjunction on the basis of the voters' views about the conjuncts. What about complex social decisions in which premises lead to the conclusion *via* inference rules other than conjunction introduction? Being right in the assessment of the truth value of a proposition is equivalent to being right in the assessment of the truth value of its negation. Since the negation of a conjunction is equivalent to the disjunction of the negated conjuncts, our results concerning conjunction introduction extend to disjunction introduction: They apply *mutatis mutandis* to social decisions in which the conclusion is a disjunction that is accepted if at least one of the disjuncts gets a majority of votes (pbp) or if the voters who accept at least one of the disjuncts are in majority (cbp).[10] But what about other inference rules, such as modus ponens, disjunctive syllogism, etc.?

(viii) What about the inference patterns in which the conclusion could be reached by a series of steps rather than in just one move? In such cases, the premise-based procedures may be applied in different ways. We could either let the voters vote on the propositions from which the main conclusion immediately follows, or instead ask them to vote on the premises that lie farther back in the inference chain. As it turns out, the premise-based procedures may deliver different results depending on the level at which it is applied. In other words, unlike the cbp, the premise-based procedures are *unstable*, as shown by the following example.

[10] Our results also apply to the disjunction introduction *via* the spbp. On the spbp, the disjunction is accepted if and only if the conjunction of the negated disjuncts is rejected, i.e., if and only if the product of the proportions of votes against the disjuncts is lower than .5.

Suppose that, in a given group, (i) there are majorities *for* each of the propositions A, B, C, D, but also (ii) majorities *against* A&B and *against* C&D. Suppose the group needs to take a stand with respect to the following proposition:

$$(X)\ (A \,\&\, B) \,\&\, (C \,\&\, D).$$

If the pbp is applied at level (i), i.e., if the vote is conducted on each of the propositions A, B, C and D, then the group will accept each of the conjuncts in X and then move on to accept X itself. But if that procedure instead is applied at level (ii), i.e., if the vote is conducted only on A&B and on C&D, then the group will reject X. The same instability will trouble the application of the spbp, if the majorities for each of A, B, C, and D are, say, larger than $\sqrt[4]{.5}$, but the majorities for each of A&B and C&D are smaller than $\sqrt{.5}$.[11] How serious are these instability problems?

(ix) How do various procedures compare from a *diachronic* perspective? It appears that such a perspective strongly favors the premise-based procedures. If new evidence comes in about one of the sub-questions, a premise-based approach can incorporate this information much more smoothly. With this approach, a new vote does not have to involve the sub-questions on which no new evidence has been received. Using the pbp or the spbp amounts to having a finer-grained diachronic truth-tracker.

(x) In our description of the spbp, we have stipulated that a conjunctive conclusion is accepted if and only if the product of the degrees of collective belief in the premises exceeds .5. This value might seem like an obvious choice if one interprets the degree of a collective belief as a probability measure: For independent conjuncts, the product of their probabilities equals the probability of the conjunction. However, from the truth-tracking perspective, it is by no means obvious that .5 is the optimal threshold value for the sophisticated premise-based procedure. A *variable* threshold of acceptance, which depends on the values of p and q and on the number of voters, might instead be preferable if one wants to maximize the chance of truth-tracking.[12]

[11] This will be possible if the majorities for each of A, B, C, and D are only marginally larger than $\sqrt[4]{.5}$ and the overlaps between the majorities for A and for B, and respectively for C and for D, are minimal. With the spbp (but not with the pbp) we could also have the opposite instability case, in which applying that procedure at level (i) would lead to rejection of X but applying it at level (ii) would lead to acceptance of X. For such an instability, it is enough if the majorities for each of A, B, C, and D are smaller than $\sqrt[4]{.5}$ but the majorities for each of A&B and C&D are larger than $\sqrt{.5}$. This will be possible if the majorities for each of A, B, C, and D are only marginally smaller than $\sqrt[4]{.5}$ and there is a maximal overlap between the majorities for A and for B, and similarly for the majorities for C and for D.

[12] We owe this observation and the previous one to Josh Snyder.

These are some of the issues that merit closer examination.

(for Luc Bovens)
Department of Philosophy, Logic and Scientific Method LSE
Houghton Street
London
WC2A 2AE
l.bovens@lse.ac.uk

(for Wlodek Rabinowicz)
Department of Philosophy
Lund University
Kungshuset, Lundagård, 222 22 Lund
Sweden
Wlodek.Rabinowicz@fil.lu.se

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